Electronic Transactions on Numerical Analysis.
Volume 6, pp. 133-152, December 1997.

ISSN 1068-9613.

ETNA
Kent State University
etna@mcs.kent.edu

MULTIGRID METHOD FOR $H(\mathrm{DIV})$ IN THREE DIMENSIONS*

R. HIPTMAIR†

Abstract. We are concerned with the design and analysis of a multigrid algorithm for $\boldsymbol{H}(\mathrm{div};\Omega)$–elliptic linear variational problems. The discretization is based on $\boldsymbol{H}(\mathrm{div};\Omega)$–conforming Raviart–Thomas elements. A thorough examination of the relevant bilinear form reveals that a separate treatment of vector fields in the kernel of the divergence operator and its complement is paramount. We exploit the representation of discrete solenoidal vector fields as $\mathbf{curl}$s of finite element functions in so-called Nédélec spaces. It turns out that a combined nodal multilevel decomposition of both the Raviart–Thomas and Nédélec finite element spaces provides the foundation for a viable multigrid method. Its Gauß–Seidel smoother involves an extra stage where solenoidal error components are tackled. By means of elaborate duality techniques we can show the asymptotic optimality in the case of uniform refinement. Numerical experiments confirm that the typical multigrid efficiency is actually achieved for model problems.

Key words. multigrid, Raviart–Thomas finite elements, Nédélec's finite elements, multilevel, mixed finite elements.

AMS subject classifications. 65N55, 65N30.

*Received May 15, 1997. Accepted for publication September 22, 1997. Communicated by J. Pasciak.
† Institut für Mathematik, Universität Augsburg (hiptmair@math.uni-augsburg.de)