Electronic Transactions on Numerical Analysis
Volume 55, 2022

Contents

Abstract.
Variational principles are very powerful tools when studying self-adjoint linear operators on a Hilbert space $\mathcal{H}$. Bounds for eigenvalues, comparison theorems, interlacing results, and monotonicity of eigenvalues can be proved easily with these characterizations, to name just a few. In this paper we consider generalizations of these principles to families of linear, self-adjoint operators depending continuously on a scalar in a real interval.

Key Words.
nonlinear eigenvalue problem, variational characterization, iterative projection methods, AMLS, quantum dots, viscoelastic damping, total least-squares problems, fluid-solid interaction

AMS Subject Classifications.
35P30, 47A52, 47A75, 47J10, 65F15, 65F17

Abstract.
In previous works, Bohemian matrices have attracted the attention of several researchers for their rich combinatorial structure, and they have been studied intensively from several points of view, including height, determinants, characteristic polynomials, normality, and stability. Here we consider a selected number of examples of upper Hessenberg and Toeplitz Bohemian matrix sequences whose entries belong to the population $P = \{0, \pm 1\}$, and we propose a connection with the spectral theory of Toeplitz matrix sequences and Generalized Locally Toeplitz (GLT) matrix sequences in order to give results on the localization and asymptotical distribution of their spectra and singular values. Numerical experiments that support the mathematical study are reported. A conclusion section ends the note in order to illustrate the applicability of the proposed tools to more general cases.

Key Words.
matrix (Bohemian, (upper) Hessenberg, Toeplitz), matrix sequence (Toeplitz, GLT), eigenvalue, singular value, spectral and singular value symbol/distribution

AMS Subject Classifications.
15B05, 15B36, 15A18, 11C20, 65F08, 65F15

92 The LSQR method for solving tensor least-squares problems.
Abdeslem H. Bentbib, Asmaa Khouia, and Hassane Sadok.

Abstract.
In this paper, we are interested in finding an approximate solution $\hat{\mathcal{X}}$ of the tensor least-squares minimization problem $\min_{\mathcal{X}} \left\|\mathcal{X} \times_1 A^{(1)} \times_2 A^{(2)} \times_3 \cdots \times_N A^{(N)} - \mathcal{G}\right\|$, where $\mathcal{G} \in \mathbb{R}^{J_1 \times J_2 \times \cdots \times J_N}$ and $A^{(i)} \in \mathbb{R}^{J_i \times I_i}$ $(i = 1, \ldots, N)$ are known and $\mathcal{X} \in \mathbb{R}^{I_1 \times I_2 \times \cdots \times I_N}$ is the unknown tensor to be approximated. Our approach is based on two steps. Firstly, we apply the CP or HOSVD decomposition to the right-hand side tensor $\mathcal{G}$. Secondly, we perform the well-known Golub-Kahan bidiagonalization for each coefficient matrix $A^{(i)}(i = 1, \ldots, N)$ to obtain a reduced tensor least-squares minimization problem. This type of equations may appear in color image and video restorations as we described below. Some numerical tests are performed to show the effectiveness of our proposed method.

Key Words.
HOSVD, CP decomposition, color image restoration, video restoration, LSQR

AMS Subject Classifications.
15A69, 65F

112 Analysis of parallel Schwarz algorithms for time-harmonic problems using block Toeplitz matrices.
Niall Bootland, Victorita Dolean, Alexander Kyriakis, and Jennifer Pestana.

Abstract.
In this work we study the convergence properties of the one-level parallel Schwarz method with Robin transmission conditions applied to the one-dimensional and two-dimensional Helmholtz and Maxwell's equations. One-level methods are not scalable in general. However, it has recently been proven that when impedance transmission conditions are used in the case of the algorithm being applied to the equations with absorption, then, under certain assumptions, scalability can be achieved and no coarse space is required. We show here that this result is also true for the iterative version of the method at the continuous level for strip-wise decompositions into subdomains that are typically encountered when solving wave-guide problems. The convergence proof relies on the particular block Toeplitz structure of the global iteration matrix. Although non-Hermitian, we prove that its limiting spectrum has a near identical form to that of a Hermitian matrix of the same structure. We illustrate our results with numerical experiments.

Key Words.
domain decomposition methods, Helmholtz equations, Maxwell equations, Schwarz algorithms, one-level methods, block Toeplitz matrices

AMS Subject Classifications.
65N55, 65N35, 65F10, 15A18, 15B05

142 Sparse mixture models inspired by ANOVA decompositions.
Johannes Hertrich, Fatima Antarou Ba, and Gabriele Steidl.

Abstract.
Inspired by the analysis of variance (ANOVA) decomposition of functions, we propose a Gaussian-uniform mixture model on the high-dimensional torus which relies on the assumption that the function that we wish to approximate can be well

explained by limited variable interactions. We consider three model approaches, namely wrapped Gaussians, diagonal wrapped Gaussians, and products of von Mises distributions. The sparsity of the mixture model is ensured by the fact that its summands are products of Gaussian-like density functions acting on low-dimensional spaces and uniform probability densities defined on the remaining directions. To learn such a sparse mixture model from given samples, we propose an objective function consisting of the negative log-likelihood function of the mixture model and a regularizer that penalizes the number of its summands. For minimizing this functional we combine the Expectation Maximization algorithm with a proximal step that takes the regularizer into account. To decide which summands of the mixture model are important, we apply a Kolmogorov-Smirnov test. Numerical examples demonstrate the performance of our approach.

Key Words.
sparse mixture models, ANOVA decomposition, wrapped Gaussian distribution, von Mises distribution, approximation of high-dimensional probability density functions, Kolmogorov-Smirnov test

AMS Subject Classifications.
62H30, 62H12, 65D15, 65C60, 62H10

169 Graph Laplacian for image deblurring.
Davide Bianchi, Alessandro Buccini, Marco Donatelli, and Emma Randazzo.

Abstract.
Image deblurring is a relevant problem in many fields of science and engineering. To solve this problem, many different approaches have been proposed, and, among the various methods, variational ones are extremely popular. These approaches substitute the original problem with a minimization problem where the functional is composed of two terms, a data fidelity term and a regularization term. In this paper we propose, in the classical non-negative constrained ℓ^2-ℓ^1 minimization framework, the use of the graph Laplacian as regularization operator. Firstly, we describe how to construct the graph Laplacian from the observed noisy and blurred image. Once the graph Laplacian has been built, we efficiently solve the proposed minimization problem by splitting the convolution operator and the graph Laplacian by the Alternating Direction Multiplier Method (ADMM). Some selected numerical examples show the good performances of the proposed algorithm.

Key Words.
image deblurring, graph Laplacian, ℓ^2-ℓ^1 regularization

AMS Subject Classifications.
65R32, 65K10, 65F22

187 Hierarchical model reduction driven by a proper orthogonal decomposition for parametrized advection-diffusion-reaction problems.
Massimiliano Lupo Pasini and Simona Perotto.

Abstract.
This work combines the Hierarchical Model (HiMod) reduction technique with a standard Proper Orthogonal Decomposition (POD) to solve parametrized partial differential equations for the modeling of advection-diffusion-reaction phenomena in elongated domains (e.g., pipes). This combination leads to what we define as HiPOD

model reduction, which merges the reliability of HiMod reduction with the computational efficiency of POD. Two HiPOD techniques are presented and assessed by an extensive numerical verification.

Key Words.
hierarchical model reduction, proper orthogonal decomposition, parametric partial differential equations, finite elements, spectral methods

AMS Subject Classifications.
65N30, 65N35, 65T40

Abstract.
We present numerical methods for computing the Schatten p-norm of positive semidefinite matrices. Our motivation stems from uncertainty quantification and optimal experimental design for inverse problems, where the Schatten p-norm defines a measure of uncertainty. Computing the Schatten p-norm of high-dimensional matrices is computationally expensive. We propose a matrix-free method to estimate the Schatten p-norm using a Monte Carlo estimator and derive convergence results and error estimates for the estimator. To efficiently compute the Schatten p-norm for non-integer and large values of p, we use an estimator using Chebyshev polynomial approximations and extend our convergence and error analysis to this setting as well. We demonstrate the performance of our proposed estimators on several test matrices and in an application to optimal experimental design for a model inverse problem.

Key Words.
Schatten p-norm, Monte Carlo estimator, optimal experimental design, Chebyshev polynomials.

AMS Subject Classifications.
65F35, 65F50, 65C05,

Abstract.
Using complex analysis, we present new error estimates for multidimensional sinc-Gauss sampling formulas for multivariate analytic functions and their partial derivatives, which are valid for wide classes of functions. The first class consists of all n-variate entire functions of exponential type satisfying a decay condition, while the second is the class of n-variate analytic functions defined on a multidimensional horizontal strip. We show that the approximation error decays exponentially with respect to the localization parameter N. This work extends former results of the first author and J. Prestin, [IMA J. Numer. Anal., 36 (2016), pp. 851–871] and [Numer. Algorithms, 86 (2021), pp. 1421–1441], on two-dimensional sinc-Gauss sampling formulas to the general multidimensional case. Some numerical experiments are presented to confirm the theoretical analysis.

Key Words.
multidimensional sinc-Gauss sampling formula, multivariate analytic function, localization operator, error estimate

AMS Subject Classifications.
94A20, 32A15, 41A25, 41A80

Abstract.
In this paper, a decoupling finite element method with different time steps for the micropolar fluid model is considered. The theoretical analysis shows that the proposed method is stable and convergent. Further, in order to show the efficiency of the method, we present some numerical results for a problem with analytical solution, and we test the method for the stirring problem of a passive scalar. From these numerical results, we can see that the method is efficient for micropolar fluid flows.

Key Words.
finite element method, decoupling method, different time steps, micropolar fluid model

AMS Subject Classifications.
65M12, 65M15, 65M60

Abstract.
We give a one-step convergence analysis of inexact Anderson acceleration for the fixed point iteration $x_{k+1} = g(x_k)$ with a potentially non-contractive mapping g, where $g(x_k)$ is evaluated approximately and the minimization of the nonlinear residual norms is performed in the vector 2-norm by the linear least-squares method. If g is non-contractive, then the original fixed point iteration does not converge, but a recent analysis by S. Pollock and L. Rebholz [IMA J. Numer. Anal., 41 (2021), pp. 2841–2872] shows that Anderson acceleration may still converge provided that the minimization at each step has a sufficient gain. In this paper, we show that inexact Anderson acceleration exhibits essentially the same convergence behavior as the exact algorithm if each $g(x_k)$ is evaluated with an error proportional to the nonlinear residual norm $\|w_k\| = \|g(x_k) - x_k\|$, regardless of whether g is contractive or not. This means that the existing relationship between exact and inexact Anderson acceleration can be generalized in a unified framework for both contractive and non-contractive mappings. Numerical experiments show that the inexact algorithm can converge as rapidly as the exact counterpart while it can lower the computational cost.

Key Words.
fixed point iteration, inexact Anderson acceleration, non-contractive mapping, one-step convergence

AMS Subject Classifications.
65N22, 65H10, 65F50

Abstract.
Biot's equations of poroelasticity contain a parabolic system for the evolution of the pressure, which is coupled with a quasi-stationary equation for the stress tensor. Thus, it is natural to extend the existing work on isogeometric space-time methods to this more advanced framework of a partial differential-algebraic equation (PDAE). A space-time approach based on finite elements has already been introduced. We present a new weak formulation in space and time that is appropriate for an isogeometric discretization and analyze its convergence properties. Our approach is based on a single variational problem and hence differs from the iterative space-time schemes considered so far. Further, it enables high-order convergence. Numerical experiments that have been carried out confirm the theoretical findings.

Key Words.
Biot's poroelasticity model, isogeometric analysis, space-time discretization, high-order convergence

AMS Subject Classifications.
76S05, 74F10, 65M12, 65M22, 65M60, 65D07

Abstract.
We propose a method for exploiting compression in computing the solution to a system of linear algebraic equations. The method is based on computing an approximate solution in a reduced space, and thus we seek a basis in which the solution has a compressed representation and can consequently be computed more efficiently. Although the method is completely general, it is especially effective for linear systems resulting from discretization of an underlying continuous problem, which will be our main focus. We address three primary issues: (1) how to compute an approximate solution to a given linear system using a given basis, (2) how to choose a basis that will yield significant compression, and (3) how to detect when the chosen basis is of sufficient dimension to provide a satisfactory approximation. While all three aspects have antecedents in previous ideas and methods, we combine, adapt, and extend them in a manner we believe to be novel for the purpose of solving discretized linear systems. We demonstrate that the resulting method can be competitive with—and often substantially outperforms—current standard methods and is effective for efficiently solving linear systems resulting from the discretization of major classes of continuous problems, including both differential equations and integral equations.

Key Words.
linear systems, compression basis, compressed solution, projection method, discretized linear system, regularization

AMS Subject Classifications.
65H10, 65N22, 65F10, 65F22

Abstract.
We consider monolithic algebraic multigrid (AMG) algorithms for the solution of block linear systems arising from multiphysics simulations. While the multigrid idea is applied directly to the entire linear system, AMG operators are constructed by leveraging the matrix block structure. In particular, each block corresponds to a set of physical unknowns and physical equations. Multigrid components are constructed by first applying existing AMG procedures to matrix sub-blocks. The resulting AMG sub-components are then composed together to define a monolithic AMG preconditioner. Given the problem-dependent nature of multiphysics systems, different blocking choices may work best in different situations, and so software flexibility is essential. We apply different blocking strategies to systems arising from resistive magnetohydrodynamics in order to demonstrate the associated trade-offs.

Key Words.
multigrid, algebraic multigrid, multiphysics, magnetohydrodynamics

AMS Subject Classifications.
68Q25, 68R10, 68U05

Abstract.
By reformulating the linear complementarity problem involving a positive definite Toeplitz matrix as an equivalent fixed-point system, we construct a modulus-based circulant and skew-circulant splitting (MCSCS) iteration method. We also analyze the convergence of the method and show that the new method is effective by providing some numerical results.

Key Words.
linear complementarity problem, Toeplitz matrix, modulus-based circulant and skew-circulant splitting

AMS Subject Classifications.
65F10, 65Y05, 65H10

Abstract.
In this article, we use the complex compensated Horner method to derive a compensated Ehrlich-Aberth method for the accurate computation of all roots, real or complex, of a polynomial. In particular, under suitable conditions, we prove that the limiting accuracy for the compensated Ehrlich-Aberth iterations is as accurate as if computed in twice the working precision and then rounded to the working precision. Moreover, we derive a running error bound for the complex compensated Horner method and use it to form robust stopping criteria for the compensated Ehrlich-Aberth iterations. Finally, extensive numerical experiments illustrate that the backward and forward errors of the root approximations computed via the compensated Ehrlich-Aberth method are similar to those obtained with a quadruple precision implementation of the Ehrlich-Aberth method with a significant speed-up in terms of computation time.

Key Words.
polynomial evaluation, error-free transformations, polynomial roots, backward error, forward error, rounding error analysis

AMS Subject Classifications.
65H04, 65Y20, 65-04

424 Error bounds for Gaussian quadrature formulae with Legendre weight function for analytic integrands.
D. R. Jandrlić, Đ. M. Krtinić, Lj. V. Mihić, A. V. Pejčev, and M. M. Spalević.

Abstract.
In this paper we are concerned with a method for the numerical evaluation of the error terms in Gaussian quadrature formulae with the Legendre weight function. Inspired by the work of H. Wang and L. Zhang [J. Sci. Comput., 75 (2018), pp. 457–477] and applying the results of S. Notaris [Math. Comp., 75 (2006), pp. 1217–1231], we determine an explicit formula for the kernel. This explicit expression is used for finding the points on ellipses where the maximum of the modulus of the kernel is attained. Effective error bounds for this quadrature formula for analytic integrands are derived.

Key Words.
Gauss quadrature formulae, Legendre polynomials, remainder term for analytic function, error bound

AMS Subject Classifications.
65D32, 65D30, 41A55

438 Decay bounds for Bernstein functions of Hermitian matrices with applications to the fractional graph Laplacian.
Marcel Schweitzer.

Abstract.
For many functions of matrices $f(A)$, it is known that their entries exhibit a rapid—often exponential or even superexponential—decay away from the sparsity pattern of the matrix A. In this paper, we specifically focus on the class of Bernstein functions, which contains the fractional powers A^α, $\alpha \in (0,1)$, as an important special case, and derive new decay bounds by exploiting known results for the matrix exponential in conjunction with the Lévy-Khintchine integral representation. As a particular special case, we find a result concerning the power law decay of the strength of connection in nonlocal network dynamics described by the fractional graph Laplacian, which improves upon known results from the literature by doubling the exponent in the power law.

Key Words.
matrix functions, Bernstein functions, off-diagonal decay, graph Laplacian, fractional powers, nonlocal dynamics

AMS Subject Classifications.
05C82, 15A16, 65F50, 65F60

455 Structured shifts for skew-symmetric matrices.
Chen Greif.

Abstract.
We consider the use of a skew-symmetric block-diagonal matrix as a structured shift. Properties of Hamiltonian and skew-Hamiltonian matrices are used to show that the shift can be effectively used in the iterative solution of skew-symmetric linear systems or nonsymmetric linear systems with a dominant skew-symmetric part. Eigenvalue analysis and some numerical experiments confirm our observations.

Key Words.
skew-symmetric matrix, structured shift, Hamiltonian matrix, skew-Hamiltonian matrix, eigenvalue analysis, iterative solution of linear systems

AMS Subject Classifications.
65F08, 65F10, 65F50, 15A12, 15B57

Abstract.
A parametric model order reduction (MOR) approach for simulating high-dimensional models arising in financial risk analysis is proposed on the basis of the proper orthogonal decomposition (POD) approach to generate small model approximations for high-dimensional parametric convection-diffusion reaction partial differential equations (PDE). The proposed technique uses an adaptive greedy sampling approach based on surrogate modeling to efficiently locate the most relevant training parameters, thus generating the optimal reduced basis. The best suitable reduced model is procured such that the total error is less than a user-defined tolerance. The three major errors considered are the discretization error associated with the full model obtained by discretizing the PDE, the model order reduction error, and the parameter sampling error. The developed technique is analyzed, implemented, and tested on industrial data of a puttable steepener under the two-factor Hull-White model. The results illustrate that the reduced model provides a significant speedup with excellent accuracy over a full model approach, demonstrating its potential for applications to the historical or Monte Carlo Value-at-Risk calculations.

Key Words.
financial risk analysis, short-rate models, convection-diffusion-reaction equation, finite element method, parametric model order reduction, proper orthogonal decomposition, adaptive greedy sampling, error analysis, packaged retail investment and insurance-based products

AMS Subject Classifications.
35L10, 65M60, 91G30, 91G60, 91G80

Abstract.
We study computational methods for computing the distance to singularity, the distance to the nearest high-index problem, and the distance to instability for linear differential-algebraic systems (DAEs) with dissipative Hamiltonian structure. While for general unstructured DAEs the characterization of these distances is very difficult and partially open, it has been shown in [C. Mehl, V. Mehrmann, and M. Wojtylak,

Distance problems for dissipative Hamiltonian systems and related matrix polynomials, Linear Algebra Appl., 623 (2021), pp. 335–366] that for dissipative Hamiltonian systems and related matrix pencils there exist explicit characterizations. We will use these characterizations for the development of computational methods to approximate these distances via methods that follow the flow of a differential equation converging to the smallest perturbation that destroys the property of regularity, index one, or stability.

Key Words.
dissipative Hamiltonian systems, structured distance to singularity, structured distance to high-index problem, structured distance to instability, low-rank perturbation, differential-algebraic system

AMS Subject Classifications.
15A18, 15A21, 65K05, 15A22

Abstract.
Subspace recycling iterative methods and other subspace augmentation schemes are a successful extension to Krylov subspace methods in which a Krylov subspace is augmented with a fixed subspace spanned by vectors deemed to be helpful in accelerating convergence or conveying knowledge of the solution. Recently, a survey was published, in which a framework describing the vast majority of such methods was proposed [Soodhalter et al., GAMM-Mitt., 43 (2020), Art. e202000016]. In many of these methods, the Krylov subspace is one generated by the system matrix composed with a projector that depends on the augmentation space. However, it is not a requirement that a projected Krylov subspace be used. There are augmentation methods built on using Krylov subspaces generated by the original system matrix, and these methods also fit into the general framework. In this note, we observe that one gains implementation benefits by considering such augmentation methods with unprojected Krylov subspaces in the general framework. We demonstrate this by applying the idea to the R^3GMRES method proposed in [Dong et al., Electron. Trans. Numer. Anal., 42 (2014), pp. 136–146] to obtain a simplified implementation and to connect that algorithm to early augmentation schemes based on flexible preconditioning [Saad, SIAM J. Matrix Anal. Appl., 18 (1997)].

Key Words.
Krylov subspaces, augmentation, recycling, discrete ill-posed problems

AMS Subject Classifications.
65F10, 65F50, 65F08

Abstract.
In theory, the Lanczos algorithm generates an orthogonal basis of the corresponding Krylov subspace. However, in finite precision arithmetic the orthogonality and linear independence of the computed Lanczos vectors is usually lost quickly. In this paper we study a class of matrices and starting vectors having a special nonzero structure that guarantees exact computations of the Lanczos algorithm whenever

floating point arithmetic satisfying the IEEE 754 standard is used. Analogous results are formulated also for an implementation of the conjugate gradient method called cgLanczos. This implementation then computes approximations that agree with their exact counterparts to a relative accuracy given by the machine precision and the condition number of the system matrix. The results are extended to the Arnoldi algorithm, the nonsymmetric Lanczos algorithm, the Golub-Kahan bidiagonalization, the block-Lanczos algorithm, and their counterparts for solving linear systems.

Key Words.
Lanczos algorithm, exact computations, finite precision arithmetic, rounding errors

AMS Subject Classifications.
65F10, 65F15

568 Analysis of stability and convergence for L-type formulas combined with a spatial finite element method for solving subdiffusion problems.
Mohadese Ramezani, Reza Mokhtari, and Gundolf Haase.

Abstract.
A time-fractional diffusion equation with the Caputo fractional derivative of order $\alpha \in (0, 1)$ is considered on a bounded polygonal domain. Some numerical methods are presented based on the finite element method (FEM) in space on a quasi-uniform mesh and L-type discretizations (i.e., L1, L1-2, and L1-2-3 formulas) to approximate the Caputo derivative. Stability and convergence of the L1-2-3 FEM as well as L1-2 FEM are proved rigorously. The lack of positivity of the coefficients of these formulas is the main difficulty in the analysis of the proposed methods. This has hampered the analysis of methods using finite elements mixed with L1-2 and L1-2-3 discretizations. Our proofs are based on the concept of a special kind of discrete Grönwall's inequality and the energy method. Numerical examples confirm the theoretical analysis.

Key Words.
subdiffusion equation, finite element method, Caputo derivative, L1 formula, L1-2 formula, L1-2-3 formula, Grönwall's inequality, stability analysis, convergence analysis

AMS Subject Classifications.
65M12, 65M60

585 Rectangular GLT sequences.
Giovanni Barbarino, Carlo Garoni, Mariarosa Mazza, and Stefano Serra-Capizzano.

Abstract.
The theory of generalized locally Toeplitz (GLT) sequences is a powerful apparatus for computing the asymptotic spectral distribution of square matrices A_n arising from the discretization of differential problems. Indeed, as the mesh fineness parameter n increases to ∞, the sequence $\{A_n\}_n$ often turns out to be a GLT sequence. In this paper, motivated by recent applications, we further enhance the GLT apparatus by developing a full theory of rectangular GLT sequences as an extension of the theory of classical square GLT sequences. We also provide two examples of application as an illustration of the potential of the theory presented herein.

Key Words.
asymptotic distribution of singular values and eigenvalues, rectangular Toeplitz matrices, rectangular generalized locally Toeplitz matrices, discretization of differential equations, finite elements, tensor products, B-splines, multigrid methods

AMS Subject Classifications.
15A18, 15B05, 47B06, 65N30, 15A69, 65D07, 65N55

618 A contribution to the conditioning of the mixed least-squares scaled total least-squares problem.
Lingsheng Meng.

Abstract.
A new closed formula for first-order perturbation estimates for the solution of the mixed least-squares scaled total least-squares (MLSSTLS) problem is presented. It is mathematically equivalent to the one by [Zhang and Wang, Numer. Algorithms, 89 (2022), pp. 1223–1246].
With this formula, new closed formulas for the relative normwise, mixed, and componentwise condition numbers of the MLSSTLS problem are derived. Compact forms and upper bounds for the relative normwise condition number are also given in order to obtain economic storage and efficient computations.

Key Words.
mixed least-squares scaled total least-squares, condition numbers, mixed least-squares total least-squares

AMS Subject Classifications.
65F20, 65F35

627 Multivariate fractal interpolation functions:
some approximation aspects and an associated fractal interpolation operator.
Kshitij Kumar Pandey and Puthan Veedu Viswanathan.

Abstract.
In the classical (non-fractal) setting, the natural kinship between theories of interpolation and approximation is well explored. In contrast to this, in the context of fractal interpolation, the interrelation between interpolation and approximation is subtle, and this duality is relatively obscure. The notion of α-fractal functions provides a proper foundation for the approximation-theoretic facet of univariate fractal interpolation functions (FIFs). However, no comparable approximation-theoretic aspects of FIFs have been developed for functions of several variables. The current article intends to open the door for intriguing interactions between approximation theory and multivariate FIFs. To this end, in the first part of this article, we develop a general framework for constructing multivariate FIFs, which is amenable to provide a multivariate analogue of an α-fractal function. Multivariate α-fractal functions provide a parameterized family of fractal approximants associated with a given multivariate continuous function. Some elementary aspects of the multivariate fractal (not necessarily linear) interpolation operator that sends a continuous function defined on a hyperrectangle to its fractal analogue are studied. As in the univariate setting, the notion of α-fractal functions serves as a basis for fractalizing various results in multivariate approximation theory, including that of multivariate splines. For our part, we provide some approximation classes of multivariate fractal functions and prove a

few results on the constrained fractal approximation of real-valued continuous functions of several variables.

Key Words.
multivariate fractal approximation, constrained approximation, fractal operator, nonlinear operator, Schauder basis, Müntz theorem

AMS Subject Classifications.
28A80, 41A05, 41A30.

652 A time-stepping finite element method for a time-fractional partial differential equation of hidden-memory space-time variable order.
Xiangcheng Zheng and Hong Wang.

Abstract.
We analyze a time-stepping finite element method for a time-fractional partial differential equation with hidden-memory space-time variable order. Due to the coupling of the space-dependent variable order with the finite element formulation and the hidden memory, the variable fractional order cannot be split from the space and destroys the monotonicity in the time-stepping discretization. Because of these difficulties, the numerical analysis of a fully-discrete finite element method of the proposed model remained untreated in the literature. We develop an alternative analysis to address these issues and to prove an optimal-order error estimate of the fully-discrete finite element scheme without any assumption on the regularity of the true solution and perform numerical experiments to substantiate the theoretical findings.

Key Words.
fractional differential equation, well-posedness and regularity, hidden memory, space-time variable order, time-stepping finite element discretization, error estimate

AMS Subject Classifications.
35R11, 65M12

671 Constructing diffeomorphisms between simply connected plane domains.
Kendall Atkinson, David Chien, and Olaf Hansen.

Abstract.
Consider a simply connected domain $\Omega \subset \mathbb{R}^2$ with boundary $\partial\Omega$ that is given by a smooth function $\varphi : [a, b] \mapsto \mathbb{R}^2$. Our goal is to calculate a diffeomorphism $\Phi : \mathbb{B}_1(0) \mapsto \Omega$, $\mathbb{B}_1(0)$ the open unit disk in $\mathbb{R}^2$. We present two different methods where both methods are able to handle boundaries $\partial\Omega$ that are not star-shaped. The first method is based on an optimization algorithm that optimizes the curvature of the boundary, and the second method is based on the physical principle of minimizing a potential energy. Both methods construct first a homotopy between the boundary $\partial\mathbb{B}_1(0)$ and $\partial\Omega$ and then extend the boundary homotopy to the inside of the domains. Numerical examples show that the method is applicable to a wide variety of domains Ω.

Key Words.
domain transformations, constructing diffeomophisms, shape blending

AMS Subject Classifications.
65D05, 49Q10

Abstract.
We present a hybrid GPU-CPU version of the AGMG software, a popular algebraic multigrid (AMG) solver which implements an aggregation-based AMG method. With the new implementation, the solution stage runs on a GPU, except operations on the coarsest grid, which are executed on a CPU. To maximize the speedup, two novel features are introduced. On the one hand, ℓ_1-Jacobi smoothing is combined with polynomial acceleration (or polynomial smoothing), leading to improved performance compared with standard ℓ_1-Jacobi smoothing, while not requiring to compute eigenvalue estimates as standard polynomial smoothing does. On the other hand, besides the K-cycle used in standard AGMG, we introduce the relaxed W-cycle, which tends to combine the advantages of the K-cycle and the standard W-cycle. Numerical results show that the new implementation inherits the robustness of the original AGMG software, while bringing significant speedups on GPUs. A comparison with AmgX, a reference AMG solver from NVIDIA, suggests that the presented hybrid GPU-CPU version of AGMG is more robust and often significantly faster in the solution stage.

Key Words.
multigrid, linear systems, iterative methods, AMG, preconditioning, parallel computing, GPU

AMS Subject Classifications.
65F10, 65N22, 65Y05, 65Y10

Abstract.
In this paper we propose new adaptive stabilised mixed finite element methods for a nonlinear elliptic boundary value problem of second order in divergence form that appears, among other applications, in magnetostatics. The method is based on a three-field formulation that is augmented with suitable residual least-squares terms arising from the constitutive and equilibrium equations and from the equation that defines the gradient as an additional unknown. We show that the resulting scheme is well posed and obtain optimal error estimates. We also develop an a posteriori error analysis of residual type and derive a simple a posteriori error indicator which is reliable and locally efficient. Finally, we include several numerical experiments that confirm the theoretical results.

Key Words.
nonlinear boundary value problem, mixed finite element, stabilisation, a posteriori error estimates, magnetostatics

AMS Subject Classifications.
65N12, 65N15, 65N30, 65N50

Abstract.
When an approximant is accurate on an interval, it is only natural to try to extend it to multi-dimensional domains. In the present article we make use of the fact that linear rational barycentric interpolants converge rapidly toward analytic and several-times differentiable functions to interpolate them on two-dimensional starlike domains parametrized in polar coordinates. In the radial direction, we engage interpolants at *conformally shifted Chebyshev nodes*, which converge exponentially for analytic functions. In the circular direction, we deploy linear rational trigonometric barycentric interpolants, which converge similarly rapidly for periodic functions but now for *conformally shifted equispaced nodes*. We introduce a variant of a tensor-product interpolant of the above two schemes and prove that it converges exponentially for two-dimensional analytic functions—up to a logarithmic factor—and with an order limited only by the order of differentiability for real functions (provided that the boundary enjoys the same order of differentiability). Numerical examples confirm that the shifts permit one to reach a much higher accuracy with significantly fewer nodes, a property which is especially important in several dimensions.

Key Words.
barycentric rational interpolation, trigonometric interpolation, Lebesgue constant, conformal maps, starlike domains

AMS Subject Classifications.
41A10, 42A15, 41A20, 65D05

Abstract.
Convergence results for optimized Schwarz methods (OSM) applied as solvers for Poisson's equation in a bounded rectangular domain with Dirichlet (physical) boundary conditions and zeroth-order (Robin) artificial transmission conditions between subdomains are presented. The analysis presented applies to a continuous formulation on an arbitrary number of subdomains with cross points. Both synchronous and asynchronous versions of OSM are discussed. Convergence theorems are presented, and it is shown numerically that the hypotheses of these theorems are satisfied for certain configurations of the subdomains. Additional numerical experiments illustrate the practical behavior of the methods discussed.

Key Words.
asynchronous iterations, optimized Schwarz methods, infinite-dimensional operator

AMS Subject Classifications.
65F10, 65N22, 65N55