

ON A CONVERSE OF LAGUERRE'S THEOREM *

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Abstract. The problem of characterizing all real sequences $\{\gamma_k\}_{k=0}^{\infty}$ with the property that if $p(x) = \sum_{k=0}^n a_k x^k$ is any real polynomial, then $\sum_{k=0}^n \gamma_k a_k x^k$ has no more nonreal zeros than $p(x)$, remains open. Recently, the authors solved this problem under the additional assumption that the sequences $\{\gamma_k\}_{k=0}^{\infty}$, with the aforementioned property, can be interpolated by polynomials. The purpose of this paper is to extend this result to certain transcendental entire functions. In particular, the main result establishes a converse of a classical theorem of Laguerre for these transcendental entire functions.

Key words. Laguerre–Pólya class, entire functions, zero distribution, multiplier sequences.

AMS subject classifications. 26C10, 30D15, 30D10.

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