

# Electronic Transactions on Numerical Analysis

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### Contents

- 1 Weak symplectic schemes for stochastic Hamiltonian equations.  
*Cristina Anton, Jian Deng, and Yau Shu Wong.*

**Abstract.**

We propose a systematic approach to construct symplectic schemes in the weak sense for stochastic Hamiltonian systems. This method is based on generating functions, so it is an extension of the techniques used for constructing high-order symplectic schemes for deterministic Hamiltonian systems. Although the developed symplectic schemes are implicit, they are comparable with the explicit weak Taylor schemes in terms of the number and the complexity of the multiple Itô stochastic integrals required. We study the convergence of the proposed symplectic weak order 2 schemes. The excellent long term performance of the symplectic schemes is verified numerically.

**Key Words.**

stochastic Hamiltonian systems, symplectic integration, numerical scheme in weak sense

**AMS Subject Classifications.**

65C30, 65P10, 60H10

- 21 A moving asymptotes algorithm using new local convex approximation methods with explicit solutions.  
*Mostafa Bachar, Thierry Estebenet, and Allal Guessab.*

**Abstract.**

In this paper we propose new local convex approximations for solving unconstrained non-linear optimization problems based on a moving asymptotes algorithm. This method incorporates second-order information for the moving asymptotes location. As a consequence, at each step of the iterative process, a strictly convex approximation subproblem is generated and solved. All subproblems have explicit global optima. This considerably reduces the computational cost of our optimization method and generates an iteration sequence. For this method, we prove convergence of the optimization algorithm under basic assumptions. In addition, we present an industrial problem to illustrate a practical application and a numerical test of our method.

**Key Words.**

geometric convergence, nonlinear programming, method of moving asymptotes, multivariate convex approximation

**AMS Subject Classifications.**

65K05, 65K10, 65L10, 90C30, 46N10

- 45 A minimal residual norm method for large-scale Sylvester matrix equations.  
*Said Agoujil, Abdeslem H. Bentbib, Khalide Jbilou, and El Mostafa Sadek.*

**Abstract.**

In this paper, we present a new method for solving large-scale Sylvester matrix equations with a low-rank right-hand side. The proposed method is an iterative method based on a projection onto an extended block Krylov subspace by minimization of the norm of the residual. The obtained reduced-order problem is solved via different direct or iterative solvers that exploit the structure of the linear operator associated with the obtained matrix equation. In particular, we use the global LSQR algorithm as iterative method for the derived low-order problem. Then, when convergence is achieved, a low-rank approximate solution is computed given as a product of two low-rank matrices, and a stopping procedure based on an economical computation of the norm of the residual is proposed. Different numerical examples are presented, and the proposed minimal residual approach is compared with the corresponding Galerkin-type approach.

**Key Words.**

extended block Krylov subspaces, low-rank approximation, Sylvester equation, minimal residual methods

**AMS Subject Classifications.**

65F10, 65F30

- 60 The Davison-Man method revisited.  
*Miloud Sadkane.*

**Abstract.**

The Davison-Man method is an iterative technique for solving Lyapunov equations where the approximate solution is updated through a matrix integral and a doubling procedure. In theory, convergence is quadratic, but, in practice, there are examples where the method stagnates and no further improvement is seen. In this work, an implementation that avoids stagnation and improves the computed solution is proposed and justified.

**Key Words.**

Davison-Man, Lyapunov equation

**AMS Subject Classifications.**

65F30, 65F10

- 70 Estimates for the bilinear form  $x^T A^{-1} y$  with applications to linear algebra problems.  
*Paraskevi Fika, Marilena Mitrouli, and Paraskevi Roupa.*

**Abstract.**

Let  $A \in \mathbb{R}^{t^p \times t^p}$  be a nonsingular matrix and  $x, y$  vectors in  $\mathbb{R}^{t^p}$ . The task of this paper is to develop efficient estimation methods for the bilinear form  $x^T A^{-1} y$  based on the extrapolation of moments of the matrix  $A$  at the point  $-1$ . The extrapolation method and estimates for the trace of  $A^{-1}$  presented in Brezinski et al. [Numer. Linear Algebra Appl., 19 (2012), pp. 937–953] are extended, and families of estimates efficiently approximating the bilinear form requiring only few matrix vector products are derived. Numerical approximations of the entries and the trace of the inverse of any real nonsingular matrix are presented and several numerical results, discussions, and comparisons are given.

**Key Words.**

extrapolation, matrix moments, matrix inverse, trace

**AMS Subject Classifications.**

65F15, 65F30, 65B05, 65C05, 65J10, 15A18, 15A45

- 90** Improved perturbation bounds for the continuous-time  $H_\infty$ -control problem.  
*Nicolai D. Christov, Mihail M. Konstantinov, and Petko Hr. Petkov.*

**Abstract.**

New local perturbation bounds for the continuous-time  $H_\infty$ -control problem are obtained, which are nonlinear functions of the data perturbations and are tighter than the existing condition number-based local bounds. These nonlinear local bounds are then incorporated into nonlocal perturbation bounds which are less conservative than the existing nonlocal perturbation estimates for the  $H_\infty$ -control problem.

**Key Words.**

$H_\infty$ -control, perturbation analysis, Riccati equations

**AMS Subject Classifications.**

93B36, 65F35, 93B35

- 100** Computing approximate (block) rational Krylov subspaces without explicit inversion with extensions to symmetric matrices.  
*Thomas Mach, Miroslav S. Pranić, and Raf Vandebril.*

**Abstract.**

It has been shown that approximate extended Krylov subspaces can be computed, under certain assumptions, without any explicit inversion or system solves. Instead, the vectors spanning the extended Krylov space are retrieved in an implicit way, via unitary similarity transformations, from an enlarged Krylov subspace. In this paper this approach is generalized to rational Krylov subspaces, which aside from poles at infinity and zero, also contain finite non-zero poles. Furthermore, the algorithms are generalized to deal with block rational Krylov subspaces and techniques to exploit the symmetry when working with Hermitian matrices are also presented. For each variant of the algorithm numerical experiments illustrate the power of the new approach. The experiments involve matrix functions, Ritz-value computations, and the solutions of matrix equations.

**Key Words.**

Krylov, extended Krylov, rational Krylov, iterative methods, rotations, similarity transformations

**AMS Subject Classifications.**

65F60, 65F10, 47J25, 15A16

- 125** An efficient deflation technique for the communication-avoiding conjugate gradient method.  
*Erin Carson, Nicholas Knight, and James Demmel.*

**Abstract.**

By fusing  $s$  loop iterations, *communication/avoiding* formulations of Krylov subspace methods can asymptotically reduce sequential and parallel communication costs by a factor of  $O(s)$ . Although a number of communication-avoiding Krylov methods have been developed, there remains a serious lack of

available communication-avoiding preconditioners to accompany these methods. This has stimulated active research in discovering which preconditioners can be made compatible with communication-avoiding Krylov methods and developing communication-avoiding methods which incorporate these preconditioners. In this paper we demonstrate, for the first time, that deflation preconditioning can be applied in communication-avoiding formulations of Lanczos-based Krylov methods such as the conjugate gradient method while maintaining an  $O(s)$  reduction in communication costs. We derive a deflated version of a communication/avoiding conjugate gradient method, which is mathematically equivalent to the deflated conjugate gradient method of Saad et al. [SIAM J. Sci. Comput., 21 (2000), pp.1909–1926]. Numerical experiments on a model problem demonstrate that the communication/avoiding formulations can converge at comparable rates to the classical formulations, even for large values of  $s$ . Performance modeling illustrates that  $O(s)$  speedups are possible when performance is communication bound. These results motivate deflation as a promising preconditioner for communication-avoiding Krylov subspace methods in practice.

**Key Words.**

Krylov subspace methods, deflation, iterative solvers, minimizing communication, sparse matrices

**AMS Subject Classifications.**

65F08, 65F10, 65F50, 65Y05, 65Y20

- 142 Self-generating and efficient shift parameters in ADI methods for large Lyapunov and Sylvester equations.

*Peter Benner, Patrick Kürschner, and Jens Saak.*

**Abstract.**

Low-rank versions of the alternating direction implicit (ADI) iteration are popular and well established methods for the numerical solution of large-scale Sylvester and Lyapunov equations. Probably the biggest disadvantage of these methods is their dependence on a set of shift parameters that are crucial for fast convergence. Here we firstly review existing shift generation strategies that compute a number of shifts before the actual iteration. These approaches come with several disadvantages such as, e.g., expensive numerical computations and the difficulty to obtain necessary spectral information or data needed to initially setup their generation. Secondly, we propose two novel shift selection strategies with the motivation to resolve these issues at least partially. Both approaches generate shifts automatically in the course of the ADI iterations. Extensive numerical tests show that one of these new approaches, based on a Galerkin projection onto the space spanned by the current ADI data, is superior to other approaches in the majority of cases both in terms of convergence speed and required execution time.

**Key Words.**

Lyapunov equation, Sylvester equation, alternating directions implicit, shift parameters

**AMS Subject Classifications.**

65F10, 65F30, 15A06

- 163 Block Gram-Schmidt downdating.

*Jesse L. Barlow.*

**Abstract.**

Given positive integers  $m$ ,  $n$ , and  $p$ , where  $m \geq n + p$  and  $p \ll n$ . A method is proposed to modify the QR decomposition of  $X \in \mathbb{R}^{m \times n}$  to produce a QR decomposition of  $X$  with  $p$  rows deleted. The algorithm is based upon the classical block Gram-Schmidt method, requires an approximation of the norm of the inverse of a triangular matrix, has  $\mathcal{O}(mnp)$  operations, and achieves an accuracy in the matrix 2-norm that is comparable to similar bounds for related procedures for  $p = 1$  in the vector 2-norm. Since the algorithm is based upon matrix-matrix operations, it is appropriate for modern cache oriented computer architectures.

**Key Words.**

QR decomposition, singular value decomposition, orthogonality, downdating, matrix-matrix operations.

**AMS Subject Classifications.**

65F25, 65F20, 65F35

**188** On the location of the Ritz values in the Arnoldi process.

*G erard Meurant.*

**Abstract.**

In this paper we give a necessary and sufficient condition for a set of complex values  $\theta_1, \dots, \theta_k$  to be the Arnoldi Ritz values at iteration  $k$  for a general diagonalizable matrix  $A$ . Then we consider normal matrices and, in particular, real normal matrices with a real starting vector. We study in detail the case  $k = 2$ , for which we characterize the boundary of the region in the complex plane where pairs of complex conjugate Ritz values are located. Several examples with computations of the boundary of the feasible region are given. Finally we formulate some conjectures and open problems for the location of the Arnoldi Ritz values in the case  $k > 2$  for real normal matrices.

**Key Words.**

Arnoldi algorithm, eigenvalues, Ritz values, normal matrices

**AMS Subject Classifications.**

65F15, 65F18, 15A18

**213** A subspace iteration for symplectic matrices.

*Alexander Malyshev, Miloud Sadkane, and Ahmed Salam.*

**Abstract.**

We study the convergence behavior of an orthogonal subspace iteration for matrices whose spectrum is partitioned into three groups: the eigenvalues inside, outside, and on the unit circle. The main focus is on symplectic matrices. Numerical experiments are provided to illustrate the theory.

**Key Words.**

symplectic matrix, subspace iteration, invariant subspace

**AMS Subject Classifications.**

15A21, 65F15

- 223 Matrix decompositions for Tikhonov regularization.  
*Lothar Reichel and Xuebo Yu.*

**Abstract.**

Tikhonov regularization is a popular method for solving linear discrete ill-posed problems with error-contaminated data. This method replaces the given linear discrete ill-posed problem by a penalized least-squares problem. The choice of the regularization matrix in the penalty term is important. We are concerned with the situation when this matrix is of fairly general form. The penalized least-squares problem can be conveniently solved with the aid of the generalized singular value decomposition, provided that the size of the problem is not too large. However, it is impractical to use this decomposition for large-scale penalized least-squares problems. This paper describes new matrix decompositions that are well suited for the solution of large-scale penalized least-square problems that arise in Tikhonov regularization with a regularization matrix of general form.

**Key Words.**

ill-posed problem, matrix decomposition, generalized Krylov method, Tikhonov regularization.

**AMS Subject Classifications.**

65F22, 65F10, 65R30

- 244 On computing maximum/minimum singular values of a generalized tensor sum.  
*Asuka Ohashi and Tomohiro Sogabe.*

**Abstract.**

We consider the efficient computation of the maximum/minimum singular values of a generalized tensor sum  $T$ . The computation is based on two approaches: first, the Lanczos bidiagonalization method is reconstructed over tensor space, which leads to a memory-efficient algorithm with a simple implementation, and second, a promising initial guess given in Tucker decomposition form is proposed. From the results of numerical experiments, we observe that our computation is useful for matrices being near symmetric, and it has the potential of becoming a method of choice for other cases if a suitable core tensor can be given.

**Key Words.**

generalized tensor sum, Lanczos bidiagonalization method, maximum and minimum singular values

**AMS Subject Classifications.**

65F10