

EXTREMAL INTERPOLATORY PROBLEM OF FEJÉR TYPE FOR ALL CLASSICAL WEIGHT FUNCTIONS*

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Abstract. Several constructive solutions of interpolating problems of Fejér, Egerváry and Turán, connected with the optimal, most economical and stable interpolation are known for Jacobi, Hermite and Laguerre orthogonal polynomials. In this paper we solve the interpolatory weighted problem of Fejér type for all positive solutions of the Pearson differential equation, which generate finite or infinite sequences of the classical orthogonal polynomials. More precisely, we establish that the Fejér problem is generic in this class of polynomials and present an elementary unified proof of this fact. Next, these results are used to establish a complete solution of the Egerváry and Turán interpolatory problem.

Key words. Fejér extremal problem, Pearson differential equation, Sturm-Liouville differential equation, classical orthogonal polynomials, Lagrange optimal interpolation, weighted stability, most economical systems.

AMS subject classifications. 41A05, 42C05, 65D05, 49K35.

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