**Electronic Transactions on Numerical Analysis.**
**Volume 39, pp. 1-21, 2012.**

**ISSN 1068-9613.**

**ETNA**
**Kent State University**
**http://etna.math.kent.edu**

# THE MR$^3$-GK ALGORITHM FOR THE BIDIAGONAL SVD*

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**Abstract.** Determining the singular value decomposition of a bidiagonal matrix is a frequent subtask in numerical computations. We shed new light on a long-known way to utilize the algorithm of multiple relatively robust representations, MR$^3$, for this task by casting the singular value problem in terms of a suitable tridiagonal symmetric eigenproblem (via the Golub–Kahan matrix). Just running MR$^3$ "as is" on the tridiagonal problem does not work, as has been observed before (e.g., by B. Großer and B. Lang [Linear Algebra Appl., 358 (2003), pp. 45–70]). In this paper we give more detailed explanations for the problems with running MR$^3$ as a black box solver on the Golub–Kahan matrix. We show that, in contrast to standing opinion, MR$^3$ *can* be run safely on the Golub–Kahan matrix, with just a minor modification. A proof including error bounds is given for this claim.

**Key words.** bidiagonal matrix, singular value decomposition, MRRR algorithm, theory and implementation, Golub–Kahan matrix

**AMS subject classifications.** 65F30, 65F15, 65G50, 15A18

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*Received November 25, 2011. Accepted January 3, 2012. Published online March 5, 2012. Recommended by M. Hochstenbach. This work was carried out while P. Willems was with the Faculty of Mathematics and Natural Sciences at the University of Wuppertal. The research was partially funded by the Bundesministerium für Bildung und Forschung, contract number 01 IH 08 007 B, within the project *ELPA—Eigenwert-Löser für Petaflop-Anwendungen*.