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ERROR ESTIMATES FOR GENERAL FIDELITIES*

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Abstract. Appropriate error estimation for regularization methods in imaging and inverse problems is of enormous importance for controlling approximation properties and understanding types of solutions that are particularly favoured. In the case of linear problems, i.e., variational methods with quadratic fidelity and quadratic regularization, the error estimation is well-understood under so-called source conditions. Significant progress for nonquadratic regularization functionals has been made recently after the introduction of the Bregman distance as an appropriate error measure. The other important generalization, namely for nonquadratic fidelities, has not been analyzed so far.

In this paper we develop a framework for the derivation of error estimates in the case of rather general fidelities and highlight the importance of duality for the shape of the estimates. We then specialize the approach for several important fidelities in imaging (L^1, Kullback-Leibler).

Key words. error estimation, Bregman distance, discrepancy principle, imaging, image processing, sparsity

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