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BLOCK FACTORIZATIONS AND QD-TYPE TRANSFORMATIONS FOR THE MR^3 ALGORITHM *

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Abstract. Factorizing symmetric tridiagonal matrices and propagating the factorizations to shifted matrices are central tasks in the MR^3 algorithm for computing partial eigensystems. In this paper we propose block bidiagonal factorizations LDL^* with 1×1 and 2×2 blocks in D as an alternative to the bidiagonal and twisted factorizations used hitherto. With block factorizations, the element growth can be reduced (or avoided altogether), which is essential for the success of the MR^3 algorithm, in particular, if the latter is used to determine the singular value decomposition of bidiagonal matrices. We show that the qd algorithm used for shifting bidiagonal factorizations, e.g., $\mathsf{LDL}^* - \tau\mathsf{I} =: \mathsf{L}^+\mathsf{D}^+(\mathsf{L}^+)^*$ can be extended to work with blocks in a mixed stable way, including criteria for determining a suitable block structure dynamically.

Key words. symmetric tridiagonal matrix, eigensystem, MRRR algorithm, block bidiagonal factorizations, qd algorithm, theory and implementation

AMS subject classifications. 65F15, 65G50, 15A18

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