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ANALYSIS OF A NON-STANDARD FINITE ELEMENT METHOD BASED ON BOUNDARY INTEGRAL OPERATORS*

CLEMENS HOFREITHER†, ULRICH LANGER‡, AND CLEMENS PECHSTEIN‡

Abstract. We present and analyze a non-standard finite element method based on element-local boundary integral operators that permits polyhedral element shapes as well as meshes with hanging nodes. The method employs elementwise PDE-harmonic trial functions and can thus be interpreted as a local Trefftz method. The construction principle requires the explicit knowledge of the fundamental solution of the partial differential operator, but only locally, i.e., in every polyhedral element. This allows us to solve PDEs with elementwise constant coefficients. In this paper we consider the diffusion equation as a model problem, but the method can be generalized to convection-diffusion-reaction problems and to systems of PDEs such as the linear elasticity system and the time-harmonic Maxwell equations with elementwise constant coefficients. We provide a rigorous error analysis of the method under quite general assumptions on the geometric properties of the elements. Numerical results confirm our theoretical estimates.

Key words. Finite elements, boundary elements, BEM-based FEM, Trefftz methods, error estimates, polyhedral meshes.

AMS subject classifications. 65N30, 65N38

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† Doctoral Program *Computational Mathematics*, Altenberger Straße 69, 4040 Linz, Austria (clemens.hofreither@dk-compmath.jku.at).

‡ Institute of Computational Mathematics, Altenberger Straße 69, 4040 Linz, Austria ({ulanger, clemens.pechstein}@numa.uni-linz.ac.at).