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ANALYSIS OF THE FINITE ELEMENT METHOD FOR TRANSMISSION/MIXED BOUNDARY VALUE PROBLEMS ON GENERAL POLYGONAL DOMAINS[*]

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Abstract. We study theoretical and practical issues arising in the implementation of the Finite Element Method for a strongly elliptic second order equation with jump discontinuities in its coefficients on a polygonal domain Ω that may have cracks or vertices that touch the boundary. We consider in particular the equation $-\operatorname{div}(A\nabla u) = f \in H^{m-1}(\Omega)$ with mixed boundary conditions, where the matrix A has variable, piecewise smooth coefficients. We establish regularity and Fredholm results and, under some additional conditions, we also establish well-posedness in weighted Sobolev spaces. When Neumann boundary conditions are imposed on adjacent sides of the polygonal domain, we obtain the decomposition $u = u_{\rm reg} + \sigma$, into a function $u_{\rm reg}$ with better decay at the vertices and a function σ that is locally constant near the vertices, thus proving well-posedness in an augmented space. The theoretical analysis yields interpolation estimates that are then used to construct improved graded meshes recovering the (quasi-)optimal rate of convergence for piecewise polynomials of degree $m \geq 1$. Several numerical tests are included.

Key words. Neumann-Neumann vertex, transmission problem, augmented weighted Sobolev space, finite element method, graded mesh, optimal rate of convergence

AMS subject classifications. 65N30, 35J25, 46E35, 65N12

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