

MAPPING DIRECTED NETWORKS*

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Abstract. We develop and test a new mapping that can be applied to directed unweighted networks. Although not a “matrix function” in the classical matrix theory sense, this mapping converts an unsymmetric matrix with entries of zero or one into a symmetric real-valued matrix of the same dimension that generally has both positive and negative entries. The mapping is designed to reveal approximate directed bipartite communities within a complex directed network; each such community is formed by two set of nodes S_1 and S_2 such that the connections involving these nodes are predominantly *from* a node in S_1 and *to* a node in S_2 . The new mapping is motivated via the concept of *alternating walks* that successively respect and then violate the orientations of the links. Considering the combinatorics of these walks leads us to a matrix that can be neatly expressed via the singular value decomposition of the original adjacency matrix and hyperbolic functions. We argue that this new matrix mapping has advantages over other, exponential-based measures. Its performance is illustrated on synthetic data, and we then show that it is able to reveal meaningful directed bipartite substructure in a network from neuroscience.

Key words. bipartivity, clustering, communities, exponential, networks, neuroscience, stickiness

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