MONOTONE CONVERGENCE OF THE LANCZOS APPROXIMATIONS TO MATRIX FUNCTIONS OF HERMITIAN MATRICES[*]

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Abstract. When A is a Hermitian matrix, the action $f(A)b$ of a matrix function $f(A)$ on a vector b can efficiently be approximated via the Lanczos method. In this note we use M-matrix theory to establish that the 2-norm of the error of the sequence of approximations is monotonically decreasing if f is a Stieltjes transform and A is positive definite. We discuss the relation of our approach to a recent, more general monotonicity result of Druskin for Laplace transforms. We also extend the class of functions to certain product type functions. This yields, for example, monotonicity when approximating $\text{sign}(A)b$ with A indefinite if the Lanczos method is performed for A^2 rather than A.

Key words. matrix functions, Lanczos method, Galerkin approximation, monotone convergence, error estimates

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