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Contents

- 1 On the role of boundary conditions for CIP stabilization of higher order finite elements. *Friedhelm Schieweck.*

Abstract.

We investigate the Continuous Interior Penalty (CIP) stabilization method for higher order finite elements applied to a convection diffusion equation with a small diffusion parameter ε . Performing numerical experiments, it turns out that strongly imposed Dirichlet boundary conditions lead to relatively bad numerical solutions. However, if the Dirichlet boundary conditions are imposed on the inflow part of the boundary in a weak sense and additionally on the whole boundary in an ε -weighted weak sense due to Nitsche then one obtains reasonable numerical results. In many cases, this holds even in the limit case where the parameter of the CIP stabilization is zero, i.e., where the standard Galerkin discretization is applied. We present an analysis which explains this effect.

Key Words.

diffusion-convection-reaction equation, finite elements, Nitsche type boundary conditions, error estimates

AMS Subject Classifications.

65N15, 65N30, 65N50

- 17 On a mixed discontinuous Galerkin method. *Mika Juntunen and Rolf Stenberg.*

Abstract.

For the model Poisson problem we study the stabilized Bassi-Rebay discontinuous Galerkin method. The method is written in a mixed formulation, in which independent and fully discontinuous basis functions are used both for the scalar unknown and its flux. The continuity requirement is imposed by Nitsche's technique [Abh. Math. Sem. Univ. Hamburg, 36 (1970/71), pp. 9–15]. In the implementation the flux is then eliminated by local condensing. We show that the method is stable and optimally convergent for all positive values of the stability parameter. We also perform an a posteriori error analysis. The theoretical results are verified by numerical computations.

Key Words.

mixed method, discontinuous Galerkin method, stability, Nitsche's method

AMS Subject Classifications.

65N30, 65N55

- 33 Analysis of the DGFEM for nonlinear convection-diffusion problems. *Miloslav Feistauer and Václav Kučera.*

Abstract.

The paper is concerned with the analysis of error estimates of the discontinuous Galerkin finite element method (DGFEM) for the numerical solution of nonstationary nonlinear convection-diffusion problems equipped with Dirichlet boundary conditions. First, the case of nonlinear diffusion as well as nonlinear convection is considered. Then, the optimal $L^\infty(L^2)$ -error estimate is discussed in the case of nonlinear convection and linear diffusion.

Key Words.

nonlinear nonstationary convection-diffusion problems, nonlinear convection, nonlinear diffusion, discontinuous Galerkin finite element method, NIPG, SIPG and IIPG versions, optimal error estimates

AMS Subject Classifications.

65M60, 76M10

- 49 Numerical study of a discrete projection method for rotating incompressible flows. *Andriy Sokolov, Stefan Turek, and Maxim A. Olshanskii.*

Abstract.

This paper presents a numerical analysis for complex 3D simulations of the Stirred Tank Reactor (STR) model by a modified discrete projection method (DPM) for rotating incompressible flow. For several prototypical configurations of the STR model, we examine the multigrid behavior for momentum and pressure Poisson subproblems for different values of the time step, the angular velocity, etc., and we give examples of the convergence behavior of the (outer) DPM scheme. For a prototypical application, we visualize the complex flow behavior by injecting sources of particle tracers into the obtained flow field to observe their mixing distribution.

Key Words.

Navier–Stokes equations, Coriolis force, discrete projection method, pressure Schur complement

AMS Subject Classifications.

35Q30, 76D05, 76D07

- 63 A comparison of four- and five-point difference approximations for stabilizing the one-dimensional stationary convection-diffusion equation. *Hans-Görg Roos and Reiner Vanselow.*

Abstract.

Some recently developed finite element stabilizations of convection-diffusion problems generate in 1D five-point difference schemes. Because there are only few results on four- and five-point schemes in the literature (in contrast to three-point schemes), we discuss some properties of such schemes with special emphasis on the choice of free parameters for a singularly perturbed problem to avoid oscillations.

Key Words.

convection-diffusion, difference scheme, stabilized finite element method

AMS Subject Classifications.

65L10, 65L12, 65L60

- 76 On the definition of the SUPG parameter. *Petr Knobloch.*

Abstract.

We consider the SUPG finite element method for two-dimensional steady scalar convection-diffusion equations and discuss a recently introduced definition of the SUPG stabilization parameter along outflow Dirichlet boundaries for problems containing interior layers.

Key Words.

convection-diffusion equations, singularly perturbed problems, boundary layers, spurious oscillations, streamline upwind/Petrov-Galerkin (SUPG) method, SOLD methods

AMS Subject Classifications.

65N30

- 90 Stabilization of local projection type applied to convection-diffusion problems with mixed boundary conditions. *Gunar Matthies, Piotr Skrzypacz, and Lutz Tobiska.*

Abstract.

We present the analysis for the local projection stabilization applied to convection-diffusion problems with mixed boundary conditions. We concentrate on the enrichment approach of the local projection methods. Optimal a-priori error estimates will be proved. Numerical tests confirm the theoretical convergence results. Moreover, the local projection stabilization leads to numerical schemes which work well for problems with several types of layers. Away from layers, the solution is captured very well.

Key Words.

stabilized finite elements, convection-diffusion

AMS Subject Classifications.

65N12, 65N30

- 106 Local projection stabilization for incompressible flows: Equal-order vs. inf-sup stable interpolation. *G. Lube, G. Rapin, and J. Löwe.*

Abstract.

A standard approach to the non-stationary, incompressible Navier-Stokes model is to split the problem into linearized auxiliary problems of Oseen type. In this paper, a unified numerical analysis for finite element discretizations using the local projection stabilization method with either equal-order or inf-sup stable velocity-pressure pairs in the case of continuous pressure approximation is presented. Moreover, a careful comparison of both variants is given.

Key Words.

incompressible flows, Oseen model, stabilized FEM, local projection stabilization

AMS Subject Classifications.

65M60, 65N15, 76M10

- 123 Interaction of incompressible flow and a moving airfoil. *Martin Růžička, Miloslav Feistauer, Jaromír Horáček, and Petr Sváček.*

Abstract.

The subject of this paper is the numerical simulation of the interaction of two-dimensional incompressible viscous flow and a vibrating airfoil. A solid airfoil with two degrees of freedom can rotate around an elastic axis and oscillate in the vertical direction. The numerical simulation consists of the finite element solution of the Navier-Stokes equations coupled with a system of ordinary differential equations describing the airfoil motion. The time-dependent computational domain and a moving grid are taken into account with the aid of the Arbitrary Lagrangian-Eulerian formulation of the Navier-Stokes equations. High Reynolds numbers require the application of a suitable stabilization of the finite element discretization. Numerical tests prove that the developed method is sufficiently accurate and robust. The results are compared with experiments.

Key Words.

aeroelasticity, Navier-Stokes equations, arbitrary Lagrangian-Eulerian formulation, finite element method, stabilization for high Reynolds numbers

AMS Subject Classifications.

65M60, 76M10, 76D05

- 134 On the evaluation of finite element sensitivities to nodal coordinates. *René Schneider and Peter K. Jimack.*

Abstract.

We present a derivation of the derivative of general systems of finite element equations with respect to the coordinates of the nodes in the underlying finite element mesh. The resulting expressions allow the systematic evaluation of such derivatives without the need to resort to automatic differentiation or the expense associated with finite difference approximations. The principal motivation for this work comes from problems in optimal design, however, other potential applications are also described. The results obtained are validated through numerical examples.

Key Words.

adjoint, discrete adjoint, sensitivity analysis, differentiation, derivatives, finite element method, FEM, moving finite elements, moving mesh, moving nodes, nodal coordinates, node movement, node positions, shape derivative

AMS Subject Classifications.

65N30, 80M10, 49Q10, 49Q12

- 145 A reduced basis method for evolution schemes with parameter-dependent explicit operators. *B. Haasdonk, M. Ohlberger, and G. Rozza.*

Abstract.

During the last decades, reduced basis (RB) methods have been developed to a wide methodology for model reduction of problems that are governed by parametrized partial differential equations (P²DEs). In particular equations of elliptic and parabolic type for linear, low degree polynomial or monotonic nonlinearities have been treated successfully by RB methods using finite element schemes. Due to the

characteristic offline-online decomposition, the reduced models often become suitable for a multi-query or real-time setting, where simulation results, such as field-variables or output estimates, can be approximated reliably and rapidly for varying parameters. In the current study, we address a certain class of time-dependent evolution schemes with explicit discretization operators that are arbitrarily parameter dependent. We extend the RB methodology to these cases by applying the *empirical interpolation* method to localized discretization operators. The main technical ingredients are: (i) generation of a *collateral reduced basis* modelling the effects of the discretization operator under parameter variations in the offline-phase and (ii) an online simulation scheme based on a numerical subgrid and localized evaluations of the evolution operator. We formulate an a-posteriori error estimator for quantification of the resulting reduced simulation error. Numerical experiments on a parametrized convection problem, discretized with a finite volume scheme, demonstrate the applicability of the model reduction technique. We obtain a parametrized reduced model, which enables parameter variation with fast simulation response. We quantify the computational gain with respect to the non-reduced model and investigate the error convergence.

Key Words.

model reduction, reduced basis methods, parameter dependent explicit operators, empirical interpolation, a-posteriori error estimates

AMS Subject Classifications.

76M12, 65M15, 35L90, 35K90, 76R99

- 162 Space adaptive finite element methods for dynamic obstacle problems. *Heribert Blum, Andreas Rademacher, and Andreas Schröder.*

Abstract.

The necessity to approximate dynamic contact problems arises in many engineering processes. Because of the local effects in the contact zone, adaptive techniques are suited to improve the finite element discretisation of such problems. In this article, the Newmark method in time and finite elements in space are used to approximate the solution numerically. A spatial error estimator is derived from the semidiscretised problem. The approach relies on an auxiliary problem, which is a variational equation. An adaptive refinement process is based on this error control. Numerical results illustrate the performance of the presented method.

Key Words.

dynamic obstacle problem, a posteriori error estimation, mesh refinement, finite element method

AMS Subject Classifications.

35L85, 65M50, 65M60

- 173 Numerical analysis of Stokes equations with improved LBB dependency. *M. Wohlmuth and M. Dobrowolski.*

Abstract.

We provide a-priori bounds with improved domain dependency for the solution of Stokes equations and the numerical error of an approximation by conforming finite element methods. The domain dependency appears primarily in terms of the LBB-constant L , and several previous works have shown that L degenerates with the

aspect ratio of the domain. We explain the LBB dependency of common a-priori bounds on Du and p and improve most of these estimates by avoiding a global inf-sup condition and assuming *locally-balanced flow*, which is in particular satisfied if $g = 0$. In this case, all error bounds on $u - u_h$ and $p - p_h$, except for $\|p - p_h\|_{L^2(\Omega)}$, prove to be completely independent of L .

Key Words.

LBB-constant, inf-sup condition, Stokes equations, a-priori estimates, finite elements

AMS Subject Classifications.

65N30, 76D07

- 190** Nitsche mortaring for parabolic initial-boundary value problems. *Bernd Heinrich and Beate Jung.*

Abstract.

This paper is concerned with a method for the numerical solution of parabolic initial-boundary value problems in two-dimensional polygonal domains Ω with or without reentrant corners. The Nitsche finite element method (as a mortar method) is applied for the discretization in space, i.e., non-matching meshes are used. For the discretization in time, the backward Euler method is employed. The rate of convergence in some H^1 -like norm and in the L_2 -norm is proved for the semidiscrete as well as for the fully discrete problem. In order to improve the accuracy of the method in the presence of singularities arising in case of non-convex domains, meshes with local grading near the reentrant corner are employed for the Nitsche finite element method. Numerical results illustrate the approach and confirm the theoretically expected convergence rates.

Key Words.

parabolic problem, corner singularity, semidiscrete finite element method, non-matching meshes, Nitsche mortaring, fully discrete method

AMS Subject Classifications.

65M60, 65N30