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SCHWARZ METHODS OVER THE COURSE OF TIME*

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To the memory of Gene Golub, our leader and friend.

Abstract. Schwarz domain decomposition methods are the oldest domain decomposition methods. They were invented by Hermann Amandus Schwarz in 1869 as an analytical tool to rigorously prove results obtained by Riemann through a minimization principle. Renewed interest in these methods was sparked by the arrival of parallel computers, and variants of the method have been introduced and analyzed, both at the continuous and discrete level. It can be daunting to understand the similarities and subtle differences between all the variants, even for the specialist.

This paper presents Schwarz methods as they were developed historically. From quotes by major contributors over time, we learn about the reasons for similarities and subtle differences between continuous and discrete variants. We also formally prove at the algebraic level equivalence and/or non-equivalence among the major variants for very general decompositions and many subdomains. We finally trace the motivations that led to the newest class called optimized Schwarz methods, illustrate how they can greatly enhance the performance of the solver, and show why one has to be cautious when testing them numerically.

Key words. Alternating and parallel Schwarz methods, additive, multiplicative and restricted additive Schwarz methods, optimized Schwarz methods.

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