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ADAPTIVE K-STEP ITERATIVE METHODS FOR NONSYMMETRIC SYSTEMS OF LINEAR EQUATIONS *

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Dedicated to Professor W. Niethammer on the occasion of his 60th birthday.

Abstract. This study is concerned with k-step methods for the iterative solution of nonsymmetric systems of real linear equations. These are generalizations of the Chebyshev (2-step) iteration, with the potential for faster convergence in cases where the spectrum of the underlying coefficient matrix is not approximated well by an ellipse. We investigate the problem of optimizing the associated (asymptotic) convergence factor with respect to a finite number of points (e.g., eigenvalue estimates obtained from using the Arnoldi process). We formulate this minimization problem as an optimization problem with constraints and propose an algorithm to compute near-best k-step parameters. The computational advantages of the Chebyshev method, such as avoidance of inner products, the implementation as an adaptive method, and the simplicity of the overall scheme, carry over to the case $k > 2$.

Key words. k-step method, Chebyshev method, adaptive implementation, polynomial iteration.

AMS subject classification. 65F10.

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