

## FIRST-ORDER SYSTEM LEAST SQUARES FOR VELOCITY-VORTICITY-PRESSURE FORM OF THE STOKES EQUATIONS, WITH APPLICATION TO LINEAR ELASTICITY\*

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**Abstract.** In this paper, we study the least-squares method for the generalized Stokes equations (including linear elasticity) based on the velocity-vorticity-pressure formulation in  $d = 2$  or 3 dimensions. The least-squares functional is defined in terms of the sum of the  $L^2$ - and  $H^{-1}$ -norms of the residual equations, which is similar to that in [7], but weighted appropriately by the Reynolds number (Poisson ratio). Our approach for establishing ellipticity of the functional does not use ADN theory, but is founded more on basic principles. We also analyze the case where the  $H^{-1}$ -norm in the functional is replaced by a discrete functional to make the computation feasible. We show that the resulting algebraic equations can be preconditioned by well-known techniques uniformly well in the Reynolds number (Poisson ratio).

**Key words.** least squares, Stokes, elasticity.

**AMS subject classifications.** 65F10, 65F30.

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\*Received October 23, 1995. Accepted for publication December 16, 1995. Communicated by T. F. Chan.

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<sup>§</sup>Program in Applied Mathematics, Campus Box 526, University of Colorado at Boulder, Boulder, CO 80309-0526. This work was sponsored by the Air Force Office of Scientific Research under grant number AFOSR-86-0126, the National Science Foundation under grant number DMS-8704169, and the Department of Energy under grant number DE-FG03-93ER25165.