

## HARMONIC RAYLEIGH–RITZ EXTRACTION FOR THE MULTIPARAMETER EIGENVALUE PROBLEM\*

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**Abstract.** We study harmonic and refined extraction methods for the multiparameter eigenvalue problem. These techniques are generalizations of their counterparts for the standard and generalized eigenvalue problem. The methods aim to approximate interior eigenpairs, generally more accurately than the standard extraction does. We study their properties and give Saad-type theorems. The processes can be combined with any subspace expansion approach, for instance a Jacobi–Davidson type technique, to form a subspace method for multiparameter eigenproblems of high dimension.

**Key words.** multiparameter eigenvalue problem, two-parameter eigenvalue problem, harmonic extraction, refined extraction, Rayleigh–Ritz, subspace method, Saad’s theorem, Jacobi–Davidson

**AMS subject classifications.** 65F15, 65F50, 15A18, 15A69

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