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FOURTH ORDER TIME-STEPPING FOR LOW DISPERSION KORTEWEG-DE VRIES AND NONLINEAR SCHRÖDINGER EQUATIONS*

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Abstract. Purely dispersive equations, such as the Korteweg-de Vries and the nonlinear Schrödinger equations in the limit of small dispersion, have solutions to Cauchy problems with smooth initial data which develop a zone of rapid modulated oscillations in the region where the corresponding dispersionless equations have shocks or blow-up. Fourth order time-stepping in combination with spectral methods is beneficial to numerically resolve the steep gradients in the oscillatory region. We compare the performance of several fourth order methods for the Korteweg-de Vries and the focusing and defocusing nonlinear Schrödinger equations in the small dispersion limit: an exponential time-differencing fourth-order Runge-Kutta method as proposed by Cox and Matthews in the implementation by Kassam and Trefethen, integrating factors, time-splitting, Fornberg and Driscoll's 'sliders', and an ODE solver in MATLAB.

Key words. exponential time-differencing, Korteweg-de Vries equation, nonlinear Schrödinger equation, split step, integrating factor

AMS subject classifications. Primary, 65M70; Secondary, 65L05, 65M20

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