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FINE STRUCTURE OF THE ZEROS OF ORTHOGONAL POLYNOMIALS, I. A TALE OF TWO PICTURES*

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Dedicated to Ed Saff on the occasion of his 60th birthday

Abstract. Mhaskar-Saff found a kind of universal behavior for the bulk structure of the zeros of orthogonal polynomials for large n. Motivated by two plots, we look at the finer structure for the case of random Verblunsky coefficients and for what we call the BLS condition: $\alpha_n = Cb^n + O((b\Delta)^n)$. In the former case, we describe results of Stoiciu. In the latter case, we prove asymptotically equal spacing for the bulk of zeros.

Key words. OPUC, clock behavior, Poisson zeros, orthogonal polynomials

AMS subject classifications. 42C05, 30C15, 60G55

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