**Electronic Transactions on Numerical Analysis.**
**Volume 25, pp. 224-258, 2006.**
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**ISSN 1068-9613.**

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# STABLE MULTIRESOLUTION ANALYSIS ON TRIANGLES FOR SURFACE COMPRESSION*

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*Dedicated to Ed Saff on the occasion of his 60th birthday*

**Abstract.** Recently we developed multiscale spaces of $C^1$ piecewise quadratic polynomials on the Powell–Sabin 6-split of a triangulation relative to arbitrary polygonal domains $\Omega \subset \mathbb{R}^2$. These multiscale bases are weakly stable with respect to the $L_2$ norm. In this paper we prove that these multiscale spaces form a multiresolution analysis for the Banach space $C^1(\overline{\Omega})$ and we show that the multiscale basis forms a strongly stable Riesz basis for the Sobolev spaces $H^s(\Omega)$ with $s \in (2, \frac{5}{2})$. In other words, the norm of a function $f \in H^s(\Omega)$ can be determined from the size of the coefficients in the multiscale representation of $f$. This property makes the multiscale basis suitable for surface compression. A simple algorithm for compression is proposed and we give an optimal a priori error bound that depends on the smoothness of the input surface and on the number of terms in the compressed approximant.

**Key words.** hierarchical bases, Powell–Sabin splines, wavelets, stable approximation by splines, surface compression

**AMS subject classifications.** 41A15, 65D07, 65T60, 41A63

*Received December 2, 2004. Accepted for publication November 30, 2005. Recommended by I. Sloan. This work is partially supported by the Flemish Fund for Scientific Research (FWO Vlaanderen) project MISS (G.0211.02), and by the Belgian Programme on Interuniversity Attraction Poles, initiated by the Belgian Federal Science Policy Office. The scientific responsibility rests with the authors.

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