

NEW CONSTRUCTIONS OF PIECEWISE-CONSTANT WAVELETS*

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Dedicated to Ed Saff on the occasion of his 60th birthday

Abstract. The classical Haar wavelet system of $L_2(\mathbb{R}^n)$ is commonly considered to be very local in space. We introduce and study in this paper piecewise-constant framelets (PCF) that include the Haar system as a special case. We show that any bi-framelet pair consisting of PCFs provides the same Besov space characterizations as the Haar system. In particular, it has *Jackson-type performance* $s_J = 1$ and *Bernstein-type performance* $s_B = 0.5$. We then construct two PCF systems that are either, in high spatial dimensions, far more local than Haar, or are as local as Haar while delivering better performance: $s_J = s_B = 1$. Both representations are computed and inverted by fast algorithms.

Key words. frames, framelets, wavelets, Haar wavelets, piecewise-constant wavelets, PCF, Besov spaces, Unitary Extension Principle

AMS subject classifications. 42C15, 42C40

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