AN INTEGRAL REPRESENTATION OF SOME HYPERGEOMETRIC FUNCTIONS*

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Dedicated to Ed Saff on the occasion of his 60th birthday

Abstract. The Euler integral representation of the \( _2F_1 \) Gauss hypergeometric function is well known and plays a prominent role in the derivation of transformation identities and in the evaluation of \( _2F_1(a, b; c; 1) \), among other applications. The general \( _{p+q}F_{p+q} \) hypergeometric function has an integral representation where the integrand involves \( _qF_p \). We give a simple and direct proof of an Euler integral representation for a special class of \( _{q+1}F_q \) functions for \( q \geq 2 \). The values of certain \( _3F_2 \) and \( _4F_3 \) functions at \( x = 1 \), some of which can be derived using other methods, are deduced from our integral formula.

Key words. \( 3F_2 \) hypergeometric functions, general hypergeometric functions, integral representation

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