

## AN INTEGRAL REPRESENTATION OF SOME HYPERGEOMETRIC FUNCTIONS\*

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*Dedicated to Ed Saff on the occasion of his 60th birthday*

**Abstract.** The Euler integral representation of the  ${}_2F_1$  Gauss hypergeometric function is well known and plays a prominent role in the derivation of transformation identities and in the evaluation of  ${}_2F_1(a, b; c; 1)$ , among other applications. The general  ${}_{p+k}F_{q+k}$  hypergeometric function has an integral representation where the integrand involves  ${}_pF_q$ . We give a simple and direct proof of an Euler integral representation for a special class of  ${}_{q+1}F_q$  functions for  $q \geq 2$ . The values of certain  ${}_3F_2$  and  ${}_4F_3$  functions at  $x = 1$ , some of which can be derived using other methods, are deduced from our integral formula.

**Key words.**  ${}_3F_2$  hypergeometric functions, general hypergeometric functions, integral representation

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