Abstract. This contribution deals with the connection of $q$-Clebsch-Gordan coefficients ($q$-CGC) of the Wigner-Racah algebra for the quantum groups $SU_q(2)$ and $SU_q(1,1)$ with certain $q$-Hahn polynomials. A comparative analysis of the properties of these polynomials and $su_q(2)$ and $su_q(1,1)$ Clebsch-Gordan coefficients shows that each relation for $q$-Hahn polynomials has the corresponding partner among the properties of $q$-CGC and vice versa. Consequently, special emphasis is given to the calculations carried out in the linear space of polynomials, i.e., to the main characteristics and properties for the new $q$-Hahn polynomials obtained here by using the Nikiforov-Uvarov approach [A. F. Nikiforov, S. K. Suslov and V. B. Uvarov, Orthogonal Polynomials in Discrete Variables, Springer-Verlag, Berlin, 1991; A. F. Nikiforov and V. B. Uvarov, Classical orthogonal polynomials in a discrete variable on non-uniform lattices, Preprint Inst. Prikl. Mat. M. V. Keldysh Akad. Nauk SSSR (in Russian), 17, Moscow, 1983] on the non-uniform lattice $x(s) = \frac{s-1}{s}$. These characteristics and properties will be important to extend the $q$-Hahn polynomials to the multiple case [J. Arvesu, $q$-Discrete Multiple Orthogonal Polynomials, in preparation]. On the other hand, the aforementioned lattice allows to recover the linear one $x(s) = s$ as a limiting case, which doesn’t happen in other investigated cases [C. Campigotto, Yu. F. Smirnov and S. G. Enikeev, $q$-Analogue of the Kravchuk and Meixner orthogonal polynomials, J. of Comput. and Appl. Math., 57 (1995), pp. 87–97; A. Del Sol Mesa and Yu. F. Smirnov, Clebsch-Gordan and Racah coefficients for $U_q(1,1)$ quantum algebra, (Discrete series) In Scattering, Reactions, Transitions in Quantum Systems and Symmetry Methods, R.M. Asherova and Yu.F. Smirnov, eds., 1991], for example in $x(s) = q^s$. This fact suggests that the $q$-analogues presented here (both from the point of view of quantum group theory and special function theory) are ‘good’ ones since all characteristics and properties, and consequently, all matrix element relations will converge to the standard ones when $q$ tends to 1.

Key words. Clebsch-Gordan coefficients, discrete orthogonal polynomials ($q$-discrete orthogonal polynomials), Nikiforov-Uvarov approach, quantum groups and algebras

AMS subject classifications. 81R50, 33D45, 33C80