DUALITY OF q-POLYNOMIALS, ORTHOGONAL ON COUNTABLE SETS OF POINTS[*]

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Abstract. We review properties of q-orthogonal polynomials, related to their orthogonality, duality and connection with the theory of symmetric (self-adjoint) operators, represented by a Jacobi matrix. In particular, we show how one can naturally interpret the duality of families of polynomials, orthogonal on countable sets of points. In order to obtain orthogonality relations for dual sets of polynomials, we propose to use two symmetric (self-adjoint) operators, representable (in some distinct bases) by Jacobi matrices. To illustrate applications of this approach, we apply it to several pairs of dual families of q-polynomials, orthogonal on countable sets, from the q-Askey scheme. For each such pair, the corresponding operators, representable by Jacobi matrices, are explicitly given. These operators are employed in order to find explicitly sets of points, on which the polynomials are orthogonal, and orthogonality relations for them.

Key words. q-orthogonal polynomials, duality, Jacobi matrix, orthogonality relations

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