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ETNA
Kent State University
etna@mcs.kent.edu

PARAMETER-UNIFORM FITTED MESH METHOD FOR SINGULARLY PERTURBED DELAY DIFFERENTIAL EQUATIONS WITH LAYER BEHAVIOR*

M.K. KADALBAJOO† AND K.K. SHARMA‡

Abstract. Boundary value problems for singularly perturbed differential difference equations containing delay with layer behavior are considered. There are a number of realistic models in the literature where one encounters BVPs for singularly perturbed differential difference equations with small delay, such as in variational problems in control theory and first exit time problems in modeling of activation of neurons. In some recent papers, the terms negative shift for 'delay' and positive shift for 'advance' are used. In this paper, a numerical method based on the fitted mesh approach to approximate the solution of these types of boundary value problems is presented. In this method the piecewise-uniform meshes are constructed and fitted to the boundary layer regions to adapt singular behavior of the operator in these narrow regions. Both the cases, layer on the left side boundary and layer on the right side boundary, are discussed. It is shown that the method composed of an upwind difference operator on the piecewise uniform mesh is parameter-uniform by establishing a robust error estimate. The effect of small delay on the boundary layer solution is shown by plotting the graphs of the solution for different delay values for several numerical examples. Numerical results in terms of maximum absolute error are tabulated to demonstrate the efficiency of the method.

Key words. fitted mesh, finite difference, singular perturbation, differential-difference equation, delay, boundary layer, action potential

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†Department of Mathematics, IIT Kanpur, India, (`kadal@iitk.ac.in`).
‡Institut de Mathématiques-MAB, INRIA Futurs-Bordeaux, Université Bordeaux 1, 351, Cours de la Libération, 33405 Talence Cedex, France (`kapil.sharma@math.u-bordeaux1.fr`).