

CONVERGENCE OF INFINITE PRODUCTS OF MATRICES AND INNER-OUTER ITERATION SCHEMES*

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Dedicated to Wilhelm Niethammer on the occasion of his sixtieth birthday.

Abstract. We develop conditions under which a product $\prod_{i=0}^{\infty} T_i$ of matrices chosen from a possibly infinite set of matrices $\mathcal{S} = \{T_j | j \in J\}$ converges. We obtain the following conditions which are sufficient for the convergence of the product: There exists a vector norm such that all matrices in \mathcal{S} are nonexpansive with respect to this norm and there exists a subsequence $\{i_k\}_{k=0}^{\infty}$ of the sequence of the nonnegative integers such that the corresponding sequence of operators $\{T_{i_k}\}_{k=0}^{\infty}$ converges to an operator which is paracontracting with respect to this norm. We deduce the continuity of the limit of the product of matrices as a function of the sequences $\{i_k\}_{k=0}^{\infty}$. But more importantly, we apply our results to the question of the convergence of inner-outer iteration schemes for solving **singular** consistent linear systems of equations, where the outer splitting is regular and the inner splitting is weak regular.

Key words. iterative methods, infinite products, contractions.

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