IMPLICIT FOR LOCAL EFFECTS AND EXPLICIT FOR NONLOCAL EFFECTS
IS UNCONDITIONALLY STABLE*

MIHAI ANITESCU†, FARANAK PAHLEVANI‡, AND WILLIAM J. LAYTON §

Abstract. A combination of implicit and explicit timestepping is analyzed for a system of ODEs motivated by
ones arising from spatial discretizations of evolutionary partial differential equations. Loosely speaking, the method
we consider is implicit in local and stabilizing terms in the underlying PDE and explicit in nonlocal and unstabilizing
terms. Unconditional stability and convergence of the numerical scheme are proved by the energy method and by
algebraic techniques. This stability result is surprising because usually when different methods are combined, the
stability properties of the least stable method plays a determining role in the combination.

Key words. unconditional stability, implicit-explicit methods, multiscale integration.

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†Argonne National Laboratory, Mathematics and Computer Science Division, 9700 South Cass Avenue, Argonne,
IL 60439, U.S.A. E-mail: anitescu@mcs.anl.gov.
‡University of Pittsburgh, Department of Mathematics, Pittsburgh, PA 15260, U.S.A. E-mail:
fap4@pitt.edu.
§University of Pittsburgh, Department of Mathematics, Pittsburgh, PA 15260, U.S.A. E-mail:
wjl@pitt.edu.