

## ON THE SHIFTED QR ITERATION APPLIED TO COMPANION MATRICES\*

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**Abstract.** We show that the shifted QR iteration applied to a companion matrix  $F$  maintains the weakly semiseparable structure of  $F$ . More precisely, if  $A_i - \alpha_i I = Q_i R_i$ ,  $A_{i+1} := R_i Q_i + \alpha_i I$ ,  $i = 0, 1, \dots$ , where  $A_0 = F$ , then we prove that  $Q_i$ ,  $R_i$  and  $A_i$  are semiseparable matrices having semiseparability rank at most 1, 4 and 3, respectively. This structural property is used to design an algorithm for performing a single step of the QR iteration in just  $O(n)$  flops. The robustness and reliability of this algorithm is discussed. Applications to approximating polynomial roots are shown.

**Key words.** companion matrices, QR factorization, QR iteration, semiseparable matrices, eigenvalues, polynomial roots.

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