

TRANSIENT BEHAVIOR OF POWERS AND EXPONENTIALS OF LARGE TOEPLITZ MATRICES*

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Abstract. The message of this paper is that powers of large Toeplitz matrices show critical behavior if and only if the L^∞ norm of the symbol is greater than one. Critical behavior means that the norms of the powers grow exponentially to infinity or that they run through a critical transient phase before decaying exponentially to zero. We summarize several known results that are relevant to the problem. Moreover, the paper contains some new results and illuminates the question from a perspective that might be new. The paper may serve as an introduction to a few basic phenomena one encounters in the field.

Key words. Toeplitz matrix, matrix power, matrix exponential, transient behavior.

AMS subject classifications. 47B35, 15A60, 65F35.

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