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A MULTIGRID ALGORITHM FOR SOLVING THE MULTI-GROUP, ANISOTROPIC SCATTERING BOLTZMANN EQUATION USING FIRST-ORDER SYSTEM LEAST-SQUARES METHODOLOGY*

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Abstract. This paper describes a multilevel algorithm for solving the multi-group, anisotropic scattering Boltzmann equation formulated with a first-order system least-squares methodology. A $P_n - h$ finite element discretization is used. The resulting angle discretization of this P_n approach does not exhibit the so-called "ray effects," but this discretization leads to a large coupled system of partial differential equations for the spatial coefficients, and, on scaling the system to achieve better approximation, the system coupling depends strongly on the material parameters. Away from the thick, low absorptive regime, a relatively robust multigrid algorithm for solving these spatial systems will be described. For the thick, low absorptive regime, where an incompressible elasticity-like equation appears, an additive/multiplicative Schwarz smoother gives substantial multigrid improvement over standard nodal smoothers. Rather than using higher-order or Raviart-Thomas finite element spaces, which lead to complicated implementation, only low-order, conforming finite elements are used. Numerical examples illustrating almost $h-$independent convergence rates and locking-free discretization accuracy will be given.

Key words. Boltzmann transport equation, first-order system least-squares, multigrid method.

AMS subject classifications. 65N30, 65N55, 65N15.

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