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PIECEWISE LINEAR WAVELET COLLOCATION, APPROXIMATION OF THE BOUNDARY MANIFOLD, AND QUADRATURE *

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Abstract. In this paper we consider a piecewise linear wavelet collocation method for the solution of boundary integral equations of order $\mathbf{r} = 0, -1$ over a closed and smooth boundary manifold. The trial space is the space of all continuous and piecewise linear functions defined over a uniform triangular grid and the collocation points are the grid points. For the wavelet basis in the trial space we choose the three-point hierarchical basis together with a slight modification near the boundary points of the global patches of parametrization. We choose three, four, and six term linear combinations of Dirac delta functionals as wavelet basis in the space of test functionals. The usual compression results apply, i.e., for N degrees of freedom, the fully populated stiffness matrix of N^2 entries can be approximated by a sparse matrix with no more than $\mathcal{O}(N[\log N]^2)$ nonzero entries. The topic of the present paper, however, is to show that the parametrization can be approximated by low order piecewise polynomial interpolation and that the integrals in the stiffness matrix can be computed by quadrature, where the quadrature rules are combinations of product integration applied to non analytic factors of the integrand and of high order Gauß rules applied to the analytic parts. The whole algorithm for the assembling of the matrix requires no more than $\mathcal{O}(N[\log N]^4)$ arithmetic operations, and the error of the collocation approximation, including the compression, the approximate parametrization, and the quadratures, is less than $\mathcal{O}(N^{-1}[\log N]^2)$. Note that, in contrast to well-known algorithms by v.Petersdorff, Schwab, and Schneider, only a finite degree of smoothness is required.

Key words. boundary integral equation of order 0 and -1, piecewise linear collocation, wavelet algorithm, approximation of parametrization, quadrature.

AMS subject classifications. 45L10, 65D32, 65R20, 65N38.

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