CONTINUOUS $\Theta$-METHODS FOR THE STOCHASTIC PANTOGRAPH EQUATION

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Abstract. We consider a stochastic version of the pantograph equation:
\[ dX(t) = \{aX(t) + bX(\eta t)\} \, dt + \{\sigma_1 + \sigma_2 X(t) + \sigma_3 X(\eta t)\} \, dW(t), \]
\[ X(0) = X_0, \]
for $t \in [0, T]$, a given Wiener process $W$ and $0 < \eta < 1$. This is an example of an Itô stochastic delay differential equation with unbounded memory. We give the necessary analytical theory for existence and uniqueness of a strong solution of the above equation, and of strong approximations to the solution obtained by a continuous extension of the $\Theta$-Euler scheme ($\Theta \in [0, 1]$). We establish $O(h)$ mean-square convergence of approximations obtained using a bounded mesh of uniform step $h$, rising in the case of additive noise to $O(h^2)$. Illustrative numerical examples are provided.

Key words. stochastic delay differential equation, continuous $\Theta$-method, mean-square convergence.

AMS subject classifications. 65C30, 65Q05.

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