

## Contents

- 1 General highly accurate algebraic coarsening. *Achi Brandt.*

**Abstract.**

General purely algebraic approaches for repeated coarsening of deterministic or statistical field equations are presented, including a universal way to gauge and control the quality of the coarse-level set of variables, and generic procedures for deriving the coarse-level set of equations. They apply to the equations arising from variational as well as non-variational discretizations of general, elliptic as well as non-elliptic, partial differential systems, on structured or unstructured grids. They apply to many types of disordered systems, such as those arising in composite materials, inhomogeneous ground flows, “twisted geometry” discretizations and Dirac equations in disordered gauge fields, and also to non-PDE systems. The coarsening can be inexpensive with low accuracy, as needed for multigrid solvers, or more expensive and highly accurate, as needed for other applications (e.g., once-for-all derivation of macroscopic equations). Extensions to non-local and highly indefinite (wave) operators are briefly discussed. The paper re-examines various aspects of algebraic multigrid (AMG) solvers, suggesting some new approaches for relaxation, for interpolation, and for convergence acceleration by recombining iterants. An application to the highly-disordered Dirac equations is briefly reviewed.

**Key Words.**

multiscale algorithms, multigrid, algebraic multigrid, AMG, nonlinear AMG, unstructured grids, coarsening, distributive coarsening, homogenization, compatible relaxation, Dirac equations.

**AMS(MOS) Subject Classifications.**

35A40, 65F10, 65K10, 65M55, 65N22, 65N55, 65Y05, 76M20.

**Files.**

vol.10.2000/pp1-20.dir/pp1-20.ps; vol.10.2000/pp1-20.dir/pp1-20.pdf;

**Forward References.**

- 21 Cache optimization for structured and unstructured grid multigrid. *Craig C. Douglas, Jonathan Hu, Markus Kowarschik, Ulrich Rde, Christian Weiss.*

**Abstract.**

Many current computer designs employ caches and a hierarchical memory architecture. The speed of a code depends on how well the cache structure is exploited. The number of cache misses provides a better measure for comparing algorithms than the number of multiplies.

In this paper, suitable blocking strategies for both structured and unstructured grids will be introduced. They improve the cache usage without changing the underlying algorithm. In particular, bitwise compatibility is guaranteed between the standard and the high performance implementations of the algorithms. This is illustrated by

comparisons for various multigrid algorithms on a selection of different computers for problems in two and three dimensions.

The code restructuring can yield performance improvements of factors of 2-5. This allows the modified codes to achieve a much higher percentage of the peak performance of the CPU than is usually observed with standard implementations.

**Key Words.**

computer architectures, iterative algorithms, multigrid, high performance computing, cache.

**AMS(MOS) Subject Classifications.**

65M55, 65N55, 65F10, 68-04, 65Y99.

**Files.**

vol.10.2000/pp21-40.dir/pp21-40.ps; vol.10.2000/pp21-40.dir/pp21-40.pdf;

**Forward References.**

- 41 A parallel AMG for overlapping and non-overlapping domain decomposition. *Gundolf Haase.*

**Abstract.**

There exist several approaches for the parallel solving of huge systems of linear equations resulting from the finite element (f.e.) discretization of 2<sup>nd</sup> order elliptic pdes. On the other hand, there exists a great demand for Algebraic Multigrid solvers (AMG) which have as input only matrix and right hand side or, as a substitute, the appropriate information per element. In this paper we propose a general parallel AMG algorithm using overlapping or non-overlapping data decompositions.

**Key Words.**

algebraic multigrid, parallel algorithms, domain decomposition, iterative solvers.

**AMS(MOS) Subject Classifications.**

65F10, 65N22, 65Y05.

**Files.**

vol.10.2000/pp41-55.dir/pp41-55.ps; vol.10.2000/pp41-55.dir/pp41-55.pdf;

**Forward References.**

- 56 Multilevel projection methods for nonlinear least-squares finite element computations. *Johannes Korsawe and Gerhard Starke.*

**Abstract.**

The purpose of this paper is to describe and study several algorithms for implementing multilevel projection methods for nonlinear least-squares finite element computations. These algorithms are variants of the full approximation storage (FAS) scheme which is widely used in nonlinear multilevel computations. The methods are derived in the framework of the least-squares mixed formulation of nonlinear second-order elliptic problems. The nonlinear variational problems on each level are handled by smoothers of Gauss-Seidel type based on a space decomposition of the finite element spaces. Finally, the different algorithms are tested and compared for a nonlinear elliptic problem arising from an implicit time discretization of a variably saturated subsurface flow model.

**Key Words.**

nonlinear elliptic problems, least-squares finite element method, nonlinear multi-level methods, multilevel projection methods, FAS scheme.

**AMS(MOS) Subject Classifications.**

65M55, 65M60.

**Files.**

vol.10.2000/pp56-73.dir/pp56-73.ps; vol.10.2000/pp56-73.dir/pp56-73.pdf;

**Forward References.**

- 74 A hybrid multigrid method for the steady-state incompressible Navier-Stokes equations. *Michael Pernice.*

**Abstract.**

Multigrid methods for solving the steady-state incompressible Navier-Stokes equations require an appropriate smoother and coarse grid solution strategy to be effective. Classical pressure-correction methods, such as SIMPLE and SIMPLER, are widely used as solvers in engineering analysis codes, but can also be used as effective multigrid smoothers. An inexact Newton method preconditioned by a linear multigrid method with a pressure-correction smoother can serve as a coarse grid solver. A hybrid nonlinear multigrid scheme based on combinations of these components is described. A standard benchmark problem is used to demonstrate the effectiveness of SIMPLER smoothing and the impact an inexact Newton coarse grid solver has on the resulting nonlinear multigrid scheme.

**Key Words.**

multigrid, Newton-Krylov methods, pressure-correction methods, incompressible fluid dynamics.

**AMS(MOS) Subject Classifications.**

65N55, 65H10.

**Files.**

vol.10.2000/pp74-91.dir/pp74-91.ps; vol.10.2000/pp74-91.dir/pp74-91.pdf;

**Forward References.**

- 92 Behavior of plane relaxation methods as multigrid smoothers. *Ignacio M. Llorente and N. Duane Melson.*

**Abstract.**

This paper contains the first published numerical results and analysis of the behavior of alternating plane relaxation methods as multigrid smoothers for cell-centered grids. The results are very satisfactory: plane smoothers work well in general and their performance improves considerably for strong anisotropies in the right direction because they effectively become exact solvers. In fact, the convergence rate decreases (improves) linearly with increasing anisotropy strength. The methods compared are plane Jacobi with damping, plane Jacobi with partial damping, plane Gauss-Seidel, plane zebra Gauss-Seidel, and line Gauss-Seidel. Based on numerical experiments and local mode analysis, the smoothing factor and cost per cycle of the different methods in the presence of strong anisotropies for Dirichlet boundary

conditions are compared. A four-color Gauss-Seidel method is found to have the best numerical and architectural properties of the methods considered in the present work. Although alternating direction plane relaxation schemes are simpler and more robust than other approaches, they are not currently used in industrial and production codes because they require the solution of a two-dimensional problem for each plane in each direction. We verify the theoretical predictions of Thole and Trottenberg that an exact solution of each plane is not necessary; in fact, a single two-dimensional multigrid cycle gives the same result as an exact solution, in much less execution time. As a result, alternating-plane smoothers are found to be highly efficient multigrid smoothers for anisotropic elliptic problems.

**Key Words.**

multigrid methods, anisotropic discrete operators, plane implicit methods, robust multigrid.

**AMS(MOS) Subject Classifications.**

65M55.

**Files.**

vol.10.2000/pp92-114.dir/pp92-114.ps; vol.10.2000/pp92-114.dir/pp92-114.pdf;

**Forward References.**

- 115      On preconditioning Schur complement and Schur complement preconditioning.  
*Jun Zhang.*

**Abstract.**

We study two implementation strategies to utilize Schur complement technique in multilevel recursive incomplete LU preconditioning techniques (RILUM) for solving general sparse matrices. The first strategy constructs a RILUM to precondition the original matrix. The second strategy solves the first Schur complement matrix using the lower level parts of the RILUM as the preconditioner. We discuss computational and memory costs of both strategies and the potential effect on grid independent convergence rate of RILUM with different implementation strategies.

**Key Words.**

sparse matrices, Schur complement, RILUM, preconditioning techniques.

**AMS(MOS) Subject Classifications.**

65F10, 65N06.

**Files.**

vol.10.2000/pp115-130.dir/pp115-130.ps;

vol.10.2000/pp115-130.dir/pp115-130.pdf;

**Forward References.**