

# Electronic Transactions on Numerical Analysis

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- 1** Revisiting the inverse field of values problem.  
*Natália Bebiano, João da Providência, Ana Nata, and João P. da Providência.*
- Abstract.**  
The field of values of a linear operator is the convex set in the complex plane comprising all Rayleigh quotients. For a given complex matrix, Uhlig proposed the inverse field of values problem: given a point inside the field of values, determine a unit vector for which this point is the corresponding Rayleigh quotient. In the present note we propose an alternative method of solution to those that have appeared in the literature. Our approach is based on the fact that the field of values can be seen as a union of ellipses under a compression to the two-dimensional case, in which case the problem has an exact solution. Refining an idea of Marcus and Pesce, we provide alternative algorithms to plot the field of values of a general complex matrix, which perform faster and more accurately than the existing ones.
- Key Words.**  
field of values, numerical range, inverse problem, generating vector, compression
- AMS Subject Classifications.**  
15A60, 47B35
- 13** Large-scale dual regularized total least squares.  
*Jörg Lampe and Heinrich Voss.*
- Abstract.**  
The total least squares (TLS) method is a successful approach for linear problems when not only the right-hand side but also the system matrix is contaminated by some noise. For ill-posed TLS problems, regularization is necessary to stabilize the computed solution. In this paper we present a new approach for computing an approximate solution of the dual regularized large-scale total least squares problem. An iterative method is proposed which solves a convergent sequence of projected linear systems and thereby builds up a highly suitable search space. The focus is on an efficient implementation with particular emphasis on the reuse of information.
- Key Words.**  
total least squares, regularization, ill-posedness, generalized eigenproblem
- AMS Subject Classifications.**  
65F15, 65F22, 65F30
- 41** Approximating optimal point configurations for multivariate polynomial interpolation.  
*Marc Van Barel, Matthias Humet, and Laurent Sorber.*
- Abstract.**  
Efficient and effective algorithms are designed to compute the coordinates of nearly

optimal points for multivariate polynomial interpolation on a general geometry. “Nearly optimal” refers to the property that the set of points has a Lebesgue constant near to the minimal Lebesgue constant with respect to multivariate polynomial interpolation on a finite region. The proposed algorithms range from cheap ones that produce point configurations with a reasonably low Lebesgue constant, to more expensive ones that can find point configurations for several two-dimensional shapes which have the lowest Lebesgue constant in comparison to currently known results.

**Key Words.**

(nearly) optimal points, multivariate polynomial interpolation, Lebesgue constant, greedy add and update algorithms, weighted least squares, Vandermonde matrix, orthonormal basis

**AMS Subject Classifications.**

41A10, 65D05, 65D15, 65E05

- 64 Variational image denoising while constraining the distribution of the residual.  
*Alessandro Lanza, Serena Morigi, Fiorella Sgallari, and Anthony J. Yezzi.*

**Abstract.**

We present a denoising method aimed at restoring images corrupted by additive noise based on the assumption that the distribution of the noise process is known. The proposed variational model uses Total Variation (TV) regularization (chosen simply for its popularity; any other regularizer could be substituted as well) but constrains the distribution of the residual to fit a given target noise distribution. The *residual distribution constraint* constitutes the key novelty behind our approach. The restored image is efficiently computed by the constrained minimization of an energy functional using an Alternating Directions Methods of Multipliers (ADMM) procedure. Numerical examples show that the novel residual constraint indeed improves the quality of the image restorations.

**Key Words.**

image denoising, variational models, image residual, probability distribution function

**AMS Subject Classifications.**

68U10, 65K10, 65F22, 62E17

- 85 Implicitly restarting the LSQR algorithm.  
*James Baglama and Daniel J. Richmond.*

**Abstract.**

The LSQR algorithm is a popular method for solving least-squares problems. For some matrices, LSQR may require a prohibitively large number of iterations to determine an approximate solution within a desired accuracy. This paper develops a strategy that couples the LSQR algorithm with an implicitly restarted Golub-Kahan bidiagonalization method to improve the convergence rate. The restart is carried out by first applying the largest harmonic Ritz values as shifts and then using LSQR to compute the solution to the least-squares problem. Theoretical results show how this method is connected to the augmented LSQR method of Baglama, Reichel, and Richmond [Numer. Algorithms, 64 (2013), pp. 263–293] in which the Krylov subspaces are augmented with the harmonic Ritz vectors corresponding to the smallest

harmonic Ritz values. Computed examples show the proposed method to be competitive with other methods.

**Key Words.**

Golub-Kahan bidiagonalization, iterative method, implicit restarting, harmonic Ritz values, large-scale computation, least-squares, LSQR, Krylov subspace

**AMS Subject Classifications.**

65F15, 15A18

- 106** Inversion of centrosymmetric Toeplitz-plus-Hankel Bezoutians.  
*Torsten Ehrhardt and Karla Rost.*

**Abstract.**

In this paper we discuss how to compute the inverse of a nonsingular, centrosymmetric Toeplitz-plus-Hankel Bezoutian  $B$  of order  $n$  and how to find a representation of  $B^{-1}$  as a sum of a Toeplitz and a Hankel matrix. Besides the known splitting property of  $B$  as a sum of two split-Bezoutians, the connection of the latter to Hankel Bezoutians of about half size is used. The fast inversion of the Hankel Bezoutians together with an inversion formula, which was the subject of a previous paper, leads us to an inversion formula for  $B^{-1}$  as a Toeplitz-plus-Hankel matrix. It also enables us to design an  $O(n^2)$  inversion algorithm.

**Key Words.**

Bezoutian matrix, Toeplitz matrix, Hankel matrix, Toeplitz-plus-Hankel matrix, matrix inversion

**AMS Subject Classifications.**

15A09, 15B05, 65F05

- 136**  $R^3$ GMRES: including prior information in GMRES-type methods for discrete inverse problems.  
*Yiqiu Dong, Henrik Garde, and Per Christian Hansen.*

**Abstract.**

Lothar Reichel and his collaborators proposed several iterative algorithms that augment the underlying Krylov subspace with an additional low-dimensional subspace in order to produce improved regularized solutions. We take a closer look at this approach and investigate a particular Regularized Range-Restricted GMRES method,  $R^3$ GMRES, with a subspace that represents prior information about the solution. We discuss the implementation of this approach and demonstrate its advantage by means of several test problems.

**Key Words.**

inverse problems, regularizing iterations, large-scale problems, prior information

**AMS Subject Classifications.**

65F22, 65F10

- 147** Rational interpolation methods for symmetric Sylvester equations.  
*Peter Benner and Tobias Breiten.*

**Abstract.**

We discuss low-rank approximation methods for large-scale symmetric Sylvester

equations. Following similar discussions for the Lyapunov case, we introduce an energy norm by the symmetric Sylvester operator. Given a rank  $n_r$ , we derive necessary conditions for an approximation being optimal with respect to this norm. We show that the norm minimization problem is related to an objective function based on the  $\mathcal{H}_2$ -inner product for symmetric state space systems. This objective function leads to first-order optimality conditions that are equivalent to the ones for the norm minimization problem. We further propose an iterative procedure and demonstrate its efficiency by means of some numerical examples.

**Key Words.**

Sylvester equations, rational interpolation, energy norm

**AMS Subject Classifications.**

15A24, 37M99

- 165** On the computation of the distance to quadratic matrix polynomials that are singular at some points on the unit circle.

*Alexander Malyshev and Miloud Sadkane.*

**Abstract.**

For a quadratic matrix polynomial, the distance to the set of quadratic matrix polynomials which have singularities on the unit circle is computed using a bisection-based algorithm. The success of the algorithm depends on the eigenvalue method used within the bisection to detect the eigenvalues near the unit circle. To this end, the QZ algorithm along with the Laub trick is employed to compute the anti-triangular Schur form of a matrix resulting from a palindromic reduction of the quadratic matrix polynomial. It is shown that despite rounding errors, the Laub trick followed, if necessary, by a simple refinement procedure makes the results reliable for the intended purpose. Several numerical illustrations are reported.

**Key Words.**

distance to instability, quadratic matrix polynomial, palindromic pencil, QZ algorithm, Laub trick

**AMS Subject Classifications.**

15A22, 65F35

- 177** Data completion and stochastic algorithms for PDE inversion problems with many measurements.

*Farbod Roosta-Khorasani, Kees van den Doel, and Uri Ascher.*

**Abstract.**

Inverse problems involving systems of partial differential equations (PDEs) with many measurements or experiments can be very expensive to solve numerically. Assuming that all experiments share the same set of receivers, in a recent paper we examined both stochastic and deterministic dimensionality reduction methods to reduce this computational burden. In the present article we consider the more general and practically important case where receivers are not shared across experiments. We propose a data completion approach to alleviate this problem. This is done by means of an approximation using an appropriately restricted gradient or Laplacian regularization, extending existing data for each experiment to the union of all receiver locations. Results using the method of simultaneous sources (SS) with the

completed data are then compared to those obtained by a more general but slower random subset (RS) method which requires no modifications.

**Key Words.**

stochastic algorithm, data completion, inverse problem, partial differential equation, many experiments, DC resistivity

**AMS Subject Classifications.**

65N21, 65C05

**197** Computing singular values of large matrices with an inverse-free preconditioned Krylov subspace method.

*Qiao Liang and Qiang Ye.*

**Abstract.**

We present an efficient algorithm for computing a few extreme singular values of a large sparse  $m \times n$  matrix  $C$ . Our algorithm is based on reformulating the singular value problem as an eigenvalue problem for  $C^T C$ . To address the clustering of the singular values, we develop an inverse-free preconditioned Krylov subspace method to accelerate convergence. We consider preconditioning that is based on robust incomplete factorizations, and we discuss various implementation issues. Extensive numerical tests are presented to demonstrate efficiency and robustness of the new algorithm.

**Key Words.**

singular values, inverse-free preconditioned Krylov Subspace Method, preconditioning, incomplete QR factorization, robust incomplete factorization

**AMS Subject Classifications.**

65F15, 65F08